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A simplified proof of a conjecture for the perturbed Gelfand equation from combustion theory

Philip Korman [a,*,1], Yi Li [b], Tiancheng Ouyang [c]

[a] *Department of Mathematical Sciences, University of Cincinnati, Cincinnati, OH 45221-0025, United States*
[b] *Department of Mathematics, California State University, Northridge, Northridge, CA 91330-8313, United States*
[c] *Department of Mathematics, Brigham Young University, Provo, UT 84602, United States*

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Abstract

For the perturbed Gelfand's equation on the unit ball in two dimensions, Y. Du and Y. Lou [5] proved that the curve of positive solutions is exactly S-shaped, for sufficiently small values of the secondary parameter. We present a simplified proof and some extensions. This problem is prominent in combustion theory, see e.g., the book of J. Bebernes and D. Eberly [1].

MSC: 35J61; 80A25

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1. Introduction

The following Dirichlet problem for the perturbed Gelfand's equation is prominent in combustion theory

$$\Delta u + \lambda e^{\frac{u}{1+\epsilon u}} = 0\,, \ \text{ for } |x| < 1\,, \ u = 0 \quad \text{when } |x| = 1\,, \tag{1.1}$$

* Corresponding author.
E-mail address: kormanp@ucmail.uc.edu (P. Korman).
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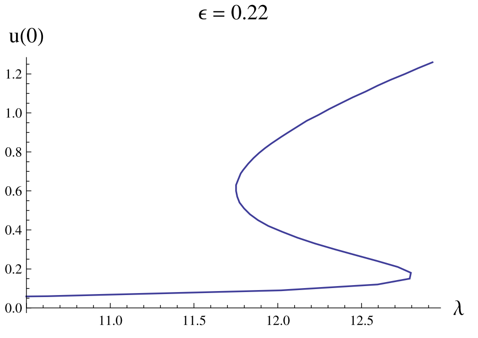

Fig. 1. An S-shaped solution curve when $\epsilon = 0.22$.

see e.g., J. Bebernes and D. Eberly [1]. Here λ and ϵ are positive parameters, and we think of λ as the primary parameter, while ϵ is the secondary, or “evolution parameter”. By the maximum principle, the solution of (1.1) is positive, and then by the classical theorem of B. Gidas, W.-M. Ni and L. Nirenberg [6] it is radially symmetric, i.e., $u = u(r)$, with $r = |x|$, and it satisfies

$$u'' + \frac{n-1}{r}u' + \lambda e^{\frac{u}{1+\epsilon u}} = 0\,, \quad 0 < r < 1\,, \quad u'(0) = u(1) = 0\,.$$

For the perturbed Gelfand’s equation on a unit ball in two dimensions

$$u'' + \frac{1}{r}u' + \lambda e^{\frac{u}{1+\epsilon u}} = 0\,, \quad 0 < r < 1\,, \quad u'(0) = u(1) = 0\,, \tag{1.2}$$

Y. Du and Y. Lou [5], building on the earlier results of P. Korman, Y. Li and T. Ouyang [12,13], proved the following theorem, thus settling a long-standing conjecture of S.V. Parter [16].

Theorem 1.1. *Suppose that $n = 2$ and $\epsilon > 0$ is sufficiently small, then solution curve $\{(\lambda, u)\}$ of (1.2) is exactly S-shaped. Moreover, the solutions lying on the upper branch and lower branch of the solution curve are asymptotically stable, while the solutions on the middle branch are unstable. (See Fig. 1.)*

Their proof was rather involved, and it was relying on some previous results of E.N. Dancer [4]. The purpose of this note is to give a simpler and self-contained proof of the S-shaped part. We also observe that some more general results can be obtained without too much extra effort. While simplifying the proof in [5], we retain several of the crucial steps from that paper: the change of variables, Lemma 4.1 and Theorem 3.1, although we generalize or simplify these results. Our new tool involves showing that the turning points are *non-degenerate*, so that they persist when the secondary parameter ϵ is varied. We also observe that computer assisted validation of bifurcation diagrams is possible for ϵ not being small. Next, we state the result in the form we prove it.

Theorem 1.2. *For ϵ sufficiently small the solution curve of (1.2) is exactly S-shaped. Moreover, at any λ where either two or three solutions occur, these solutions are strictly ordered. (See Fig. 1.)*

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