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Joint production–inventory–location problem with multi-variate normal demand

Mehrdad Shahabi[a], Amirmahdi Tafreshian[a], Avinash Unnikrishnan[b,*], Stephen D. Boyles[c]

[a] Department of Civil and Environmental Engineering, University of Michigan, 2350 Hayward, 1036 GG Brown, Ann Arbor, MI 48109, USA
[b] Department of Civil and Environmental Engineering, Engineering Building, 301D, 1930 SW 4th Avenue, Portland State University, Portland, OR 97201, USA
[c] Department of Civil, Architectural and Environmental Engineering, The University of Texas at Austin, 301 E. Dean Keeton St. Stop C1761, Austin, TX 78712-1172, USA

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ABSTRACT

This paper develops a mixed integer nonlinear programming formulation for the production–inventory–location problem with correlated demands across the retailers. Several structural results for special cases of the problem are derived and studied. A solution method based on the outer approximation of the nonlinear terms is developed to solve the problem. The efficiency of the proposed model and solution approach is investigated through extensive numerical studies. Ignoring correlations can increase the total costs of a production–location–inventory system. Accounting for correlations may lead to changes in supply chain configuration. The effect of capacity on computational times was more pronounced in lower correlations than higher correlations. In addition, we show that the efficiency of the solution method increases significantly for two special cases – when all products have the same holding cost and when the number of orders for different products at each warehouse is constrained to be the same.

1. Introduction

Effective supply chain operation and design decisions are critical to the success of many industries, given the increasingly uncertain nature of the operating environment (Friesz et al., 2011). Traditionally researchers have adopted a sequential approach in solving the strategic and tactical decisions associated with the supply chain. In the sequential approach, strategic or longer-term decisions such as where to locate facilities are taken separately from short-term tactical decisions involving inventory management. Over the last decade, several researchers have shown that such a sequential approach led to sub-optimal supply chain design and management decisions and proposed the joint location-inventory problem (Daskin et al., 2002; Shen et al., 2003; Miranda and Garrido, 2004). In the joint location–inventory framework, a single optimization model is used to determine both strategic location and tactical inventory control decisions.

Safety stock is often used as a risk pooling strategy in joint location–inventory problems to protect against retailer demand uncertainties. A majority of the works in the joint location–inventory literature assume independent retailer demand

* Corresponding author.
E-mail addresses: mshahabi@umich.edu (M. Shahabi), atafresh@umich.edu (A. Tafreshian), uavinash@pdx.edu (A. Unnikrishnan), sboyles@mail.utexas.edu (S.D. Boyles).

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(Daskin et al., 2002; Shen et al., 2003; Miranda and Garrido, 2004; Vidyarthi et al., 2007; Park et al., 2010; Hua and Willems, 2016). Some studies show that demand can be highly correlated due to the recent developments in information sharing, the increase in online purchasing, and factors such as retailer competition, cross-selling, etc. (Liu et al., 2000; Raghunathan, 2003; Zhang et al., 2011; Ganesh et al., 2014). Several studies have shown that accounting for demand correlation is critical for developing successful supply chain management strategies (Güllü, 1998; Raghunathan, 2003; So and Zheng, 2003; Helper et al., 2010; Zhang et al., 2011; Ganesha et al., 2014; Ahmadzadeh and Vahdani, 2017).

Recently Shahabi et al. (2014) demonstrated how accounting for demand correlation in safety stock cost can change the location–allocation decisions of three-echelon supply chain. However, the impact of demand correlation on daily warehouse inventory planning and plant production decisions requires further investigation. For instance, demand correlation can impose restrictions on order quantity and plant production decisions since both warehouse and plants are operating based on a limited capacity. Therefore, ignoring the existence of demand correlation would lead to significant changes in key decisions of an integrated supply chain. The goal of this work is to examine the impact of demand correlation on production, inventory, location and allocations decisions of a multi-echelon supply chain simultaneously. In particular, we consider a three level supply chain comprising of plants, warehouses, and retailers. The retailers have uncertain and correlated demand for multiple products. A mixed integer nonlinear formulation is developed to determine: (i) locations of multiple capacitated plants and warehouses, (ii) allocation of plants to warehouses and warehouses to retailers, (iii) optimal inventory levels and safety stock of inventories of multiple products at warehouses, and (iv) total amount of production by each plant. An outer approximation algorithm (OA) is proposed and customized to handle the mixed integer nonlinear formulation and deliver globally optimal solutions. This article also identifies two specific cases where the optimal order quantity is obtained analytically, resulting in significant computational savings.

2. Literature review

Daskin et al. (2002) were among the first to study the uncapacitated joint location–inventory problem. A nonlinear integer programming formulation was provided to minimize location, transportation, and inventory costs and solved using several Lagrangian relaxation-based heuristics. Shen et al. (2003) adopted a set covering formulation for Daskin et al. (2002) joint location–inventory framework and provided a column generation algorithm for two specific cases with demand variance either proportional to the mean or zero. Shen (2005) extended Daskin et al. (2002) to consider multiple commodities. Shu et al. (2005) further developed the Shen et al. (2003) column generation algorithm for general cases of demand variance. Snyder et al. (2007) studied a scenario based stochastic variant of Daskin et al. (2002). The authors focused on a specific case where the ratio of variance to the mean of demand at each retailer was the same for each scenario and solved the problem using a Lagrangian relaxation heuristic embedded in a branch and bound scheme. Shu and Sun (2006) adopted a column generation approach to solve the scenario based stochastic variant for the general case. Miranda and Garrido (2004), Miranda and Garrido (2006), and Ozsen et al. (2008) studied the capacitated variant of Daskin et al. (2002) and Shen et al. (2003) with subtle differences in the way capacity constraints were enforced. Miranda and Garrido (2004) considered throughput capacity only. Miranda and Garrido (2006) considered two types of capacity constraints – maximum order quantity and maximum inventory levels. Ozsen et al. (2008) had the most conservative capacity restriction enforcing the sum of order quantity, safety stock, and lead time demand to be less than the capacity of the warehouse. Ozsen et al. (2009) studied the multiple sourcing variant of Ozsen et al. (2008). All four papers – Miranda and Garrido (2004), Miranda and Garrido (2006), Ozsen et al. (2008), and Ozsen et al. (2009) – adopted a Lagrangian relaxation heuristic solution method. An improved Lagrangian relaxation heuristic was later introduced by Diabat et al. (2015) for handling larger size networks. Similarly, Puga and Tancrez (2017) developed a heuristic for solving large scale joint inventory location problems. All the works mentioned above assumed independent retailer demand. Atamtürk et al. (2012) used a mixed integer conic quadratic framework to solve the joint location–inventory problem with demand correlations efficiently. This paper focuses on a three level supply chain network whereas Atamtürk et al. (2012) studied a two level supply chain network.

Other researchers have extended the two level joint location–inventory problem to a multiple echelon setting (Vidyarthi et al., 2007; Park et al., 2010; Shahabi et al., 2013). Tancrez et al. (2012) adopted a continuous approach and developed an approximation heuristic to solve the three level joint location–inventory problem. Vidyarthi et al. (2007) proposed a nonlinear integer formulation for the multiple product three level production–inventory–location problem with stochastic demands. This paper differentiates itself from Vidyarthi et al. (2007) in two aspects. First, we consider correlation in retailer demands. Second, Vidyarthi et al. (2007) used a piecewise linear approximation of the nonlinear safety stock costs and solved the resulting mixed integer linear approximation using Lagrangian relaxation whereas we adopt an outer approximation algorithm which guarantees global optimality. Park et al. (2010) studied a three level supply chain network with independent retailer demand and used a Lagrangian relaxation-based heuristic. Shahabi et al. (2014) extended the formulation of Park et al. (2010) by considering demand correlations and used an outer approximation algorithm to solve the model globally. The problem studied in this paper is a generalization of the model studied in Park et al. (2010) and Shahabi et al. (2014) in the following ways:

- Our formulation determines the optimal production at each plant making it a joint production–location–inventory problem.
- The proposed formulation considers multiple products with retailer demand correlations and capacities at plants.