Applied Mathematical Modelling 46 (2017) 618–630

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Optimal inventory policy under power demand pattern and partial backlogging

Luis A. San-José[a,*], Joaquín Sicilia[b], Manuel González-De-la-Rosa[c], Jaime Febles-Acosta[c]

[a] IMUVA, Departamento de Matemática Aplicada, Universidad de Valladolid, Valladolid, Spain
[b] Departamento de Matemáticas, Estadística e Investigación Operativa, Universidad de La Laguna, Tenerife, Spain
[c] Departamento de Dirección de Empresas e Historia Económica, Universidad de La Laguna, Tenerife, Spain

ARTICLE INFO

Article history:
Received 3 February 2015
Revised 16 January 2017
Accepted 30 January 2017
Available online 7 February 2017

Keywords:
Inventory models
Power demand pattern
Partial backlogging
Shortage costs

ABSTRACT

In this paper, we study an inventory model with a power demand pattern that allows shortages. It is assumed that only a fraction of demand is backlogged during the shortage period and the remainder is considered lost sales. The aim of the paper is to determine the lot size and the length of the inventory cycle that maximize the total inventory profit per unit time. A general approach to obtain the optimal solution of the inventory problem and the maximum associated profit is developed. Some inventory models proposed in the literature are particular cases of the model analyzed here. Numerical examples are included to complement the theoretical results.

1. Introduction

The growth of commercial activities and the development of manufacturing industries have prompted the study of inventory management to fill customer orders. The maintenance and replenishment of inventories requires an exhaustive analysis of when to replenish the inventory and how much to order. Mathematical models and optimization techniques can be used to answer these questions since they contribute to the efficient management of inventory systems.

The classical economic order quantity (EOQ) model assumes that shortages are not allowed. In practice, it may be economically beneficial to have an inventory management that allows the system to be out of stock when customer demand exists. As is well-known, in the area of marketing and logistic business, stockout might generate different effects depending on the type of good and the relevance of its utility for the customer. In such a situation, demand can be backordered until a new order arrives to the inventory system. Thus, companies may consider having a backlog of orders although a certain backorder cost will be incurred. In these systems, the backorder cost can be made up of a fixed cost, and a variable cost which is proportional to the time period during the backorder. Inventory management looks at the holding cost, ordering cost and backordering cost. The total inventory profit is the difference between the revenues and the sum of these inventory costs. This profit could be maximized if planned shortages are assumed. If the sum of the ordering and holding costs is significantly greater than the shortage cost, then permitting shortages may be a good business practice because total inventory profit could be incremented without losing business.

* Corresponding author.
E-mail addresses: augusto@mat.uva.es (L.A. San-José), jsicilia@ull.es (J. Sicilia), mgonzale@ull.es (M. González-De-la-Rosa), jfebles@ull.es (J. Febles-Acosta).

http://dx.doi.org/10.1016/j.apm.2017.01.082
0307-904X/

In the above scenario, customers are willing to wait for the new product procurement and all backorders are met when a new replenishment arrives to the inventory system (full backorders). Another case is possible: demands on the system during stockouts may be lost. In that situation, the customers prefer to cancel their orders and generate lost sales. Hadley and Whitin [1] commented that, in the lost sales case, it is no longer true that annual revenue received will be independent of the operating doctrine of the vendor, that is, independent of the inventory policy considered. Thus, following this argument, we assume that the lost sale cost includes a fixed cost and a cost which is proportional to the length of time that the system is out of stock. This structure of linear costs, with a fixed part and a time dependent variable part, is also assumed in San-José et al. [2], Sicilia et al. [3] and San-José et al. [4].

These two stockout inventory models have traditionally been considered in inventory management. However, in many real inventory systems, demand can be partially backlogged and only a percentage of unfilled demand will be met when a procurement arrives. This situation of partial backordering is analyzed in this paper and requires an inventory model that represents a combination of the previous two cases. Some inventory models that assume a mixture of backorders and lost sales have also been developed by several authors (see, for instance, Chu and Chung [5], Padmanabhan and Vrat [6], Pentico and Drake [7], San-José et al. [2], [8] and [9], and Hasanov et al. [10]). A survey of deterministic EOQ models with partial backlogging is developed by Pentico and Drake [11].

How are customer demands distributed throughout the inventory cycle? The classical EOQ model assumes that demand is constant and uniformly distributed along the period. However, this assumption is often not true in practice. Thus, inventory models could be more useful where demand is not constant, but instead varies with time. Inventory systems where the demand rate is time-dependent have engaged the attention of researchers in this area. Thus, Ritchie [12] analyzed an inventory model with linear increasing demand. Bose et al. [13] developed an EOQ model with a linear positive trend in demand for deteriorating goods, allowing backlogging. Wu [14] studied an EOQ model considering shortages, time-varying demand and a process of spoilage or deterioration of products. Yang et al. [15] developed an inventory system with non-linear time-dependent demand. Sakaguchi [16] studied an inventory system where demand rate varies with time. Omar and Yeo [17] analyzed a model for an inventory system under a time-varying demand process. Mishra and Singh [18] developed a deteriorating inventory model with time-dependent demand and partial backlogging.

The assumption of time-dependent demand requires the consideration of when customers place their orders. Is demand higher at the beginning of the replenishment cycle or at the end? There are several ways to withdraw inventory quantities to meet demand. The demand patterns represent the different ways in which demand occurs during a period of time. Thus, the uniform demand pattern assumes a constant demand rate throughout the period. However, it is possible to analyze other ways of taking out units of inventory.

Once the replenishment is received, the company decides when the demand required by the loyal customers is satisfied. In some cases, it may be interested in maintaining the maximum quantity of available stock to thereby meet the demands of new customers because, otherwise, those clients could be lost due to stockout. In that case, the loyal customers are served when the inventory cycle is advanced and we know that a new replenishment will soon come. In this situation, the net inventory level could be in line with an inventory system with potential demand pattern in which the index is lower than one.

In other cases, the demand of loyal customers is provided in the first days after the receipt of the new goods. Thus, a larger quantity of goods is delivered during those first days. Consequently, the net inventory level will decrease quickly at the beginning of the inventory period. This will reduce the storage costs and liberate space in the warehouse. Now, the net inventory level could be in line with an inventory system with a potential demand pattern in which the index is greater than one.

Naddor [19] introduced the inventory models with power demand pattern. Since then, several papers have considered a power demand pattern in the inventory models. For instance, Goel and Aggarwal [20] studied an inventory model for deteriorating items with power demand pattern. Datta and Pal [21] developed the optimal inventory policy for a system with variable rate of deterioration and power demand pattern. Lee and Wu [22] analyzed an inventory system for items with Weibull distributed deterioration, assuming shortages completely backlogged and power demand pattern. Later on, Dye [23] studied an inventory model with power demand pattern and time-proportional backlogging rate. More recently, Singh et al. [24] studied an EOQ model for items with variable rate of deterioration, power demand and backlogging inversely proportional to the waiting time for the next replenishment. Rajeswari and Vanjikkodi [25] developed an inventory model with power demand, constant deterioration and variable backlogging rate which is dependent on the length of the waiting time. Mishra and Singh [26] analyzed an EOQ model for perishable items with quadratic deterioration, assuming shortages partially backlogged and power demand pattern.

This last group of papers on inventory systems, where demand follows a power pattern, assumed that the inventory cycle was fixed. Thus, in the development of these inventory models, the total inventory cost per unit time is a function that depends on a single decision variable. However, the inventory cycle in our paper is not fixed and the total inventory profit per unit time depends on two decision variables: the length of the inventory cycle and the maximum stock level. Sicilia et al. [27] studied two inventory systems with power demand pattern, one considering that demands during the stockout period are lost sales and another supposing that the total demand during the stockout period is backordered. In this paper, we extend both inventory models analyzed by those authors. The main contribution of this paper is the detailed description of the optimal inventory policies obtained for the inventory system with a power demand pattern and partially backlogged shortages, assuming a structure of linear shortage cost.