Mechanical Systems and Signal Processing 96 (2017) 31–44

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

A stochastic harmonic function representation for non-stationary stochastic processes

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ARTICLE INFO

Article history:
Received 11 December 2016
Received in revised form 17 March 2017
Accepted 31 March 2017

Keywords:
Stochastic harmonic function
Non-stationary stochastic processes
Evolutionary power spectral density
Spectral representation method
Wavelet-based EPSD estimation

ABSTRACT

The time-domain representation of non-stationary stochastic processes is of paramount importance, in particular for response analysis and reliability evaluation of nonlinear structures. In the present paper a stochastic harmonic function (SHF) representation originally developed for stationary processes is extended to evolutionary non-stationary processes. Utilizing the new scheme, the time-domain representation of non-stationary stochastic processes is expressed as the linear combination of a series of stochastic harmonic components. Different from the classical spectral representation (SR), not only the phase angles but also the frequencies and their associated amplitudes, are treated as random variables. The proposed method could also be regarded as an extension of the classical spectral representation method. However, it is rigorously proved that the new scheme well accommodates the target evolutionary power spectral density function. Compared to the classical spectral representation method, moreover, the new scheme needs much fewer terms to be retained. The first four moments and the distribution properties, e.g., the asymptotical Gaussianity, of the simulated stochastic process via SHF representation are studied. Numerical examples are addressed for illustrative purposes, showing the effectiveness of the proposed scheme.

1. Introduction

Most engineering dynamic actions exhibit large degree of variations, and shall be regarded as stochastic processes. For instance, nearly 70 years ago Housner recognized the randomness of earthquake ground accelerations, and proposed the white noise model [9]. About ten years later, it was well accepted that the earthquake ground motions should be regarded as stochastic processes, not white noise but with some dominant frequency content which could be captured by the power spectral density (PSD) [12,33]. Since then the modification and improvement of the PSD model for earthquake ground motions have been extensively studied [10,3]. Meanwhile, the PSD models for wind and sea waves were also investigated [4,27]. Very soon the non-stationary properties of ground motion accelerations were well recognized [35,1]. Almost simultaneously, the theory of evolutionary power spectral density (EPSD) was timely proposed [28]. It was successfully adopted in many investigations associated with the characterization of earthquake ground motions [19,18,36,30].

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http://dx.doi.org/10.1016/j.ymssp.2017.03.048
0888-3270/

It is well known that in stochastic dynamics of linear structures an elegant relationship between the input and output PSD exists for both stationary and non-stationary excitations [22,15]. Unfortunately, due to the unclosure property such relationship could not be extended to nonlinear systems, i.e., even the PSD or EPSD of input is known, the EPSD of output could not be obtained directly by a spectral analysis in frequency domain [22]. As a matter of fact, for stochastic dynamic analysis of nonlinear structures, time domain analysis is generally the most appropriate method. Consequently, the time domain representation of non-stationary stochastic processes, i.e., to represent them by explicit functions of a set of basic random variables, is necessary. For this purpose, the major requirements include: (1) the represented non-stationary processes could capture the statistical properties, e.g., the PSD or EPSD, of the target processes in high accuracy; and (2) the number of basic random variables should be as small as possible, because the increase of random variables will induce huge difficulty in the stochastic response analysis of the system. For instance, many prevalent methods, including the extensively studied orthogonal polynomial expansion method [7,14,6], are greatly challenged by the increase of number of random variables. Even in the Monte Carlo method, it is hard to avoid strong correlation of pseudo-random number vector in high dimensions [8].

In the past decades, a variety of approaches have been developed for the above purpose. Generally, the developed approaches could be classified into physically based approaches and mathematical expansion based approaches. In the approaches belonging to the former class, the physical principle/mechanism embedded in the random phenomena was invoked explicitly to obtain a function of temporal arguments and basic random variables. These basic random variables are usually physically meaningful, and their distributions could be determined by measured data, say the earthquake ground acceleration records [34,26], fluctuating wind speed data [16] or sea wave data [21]. Generally, very few random variables are needed in this way to capture the major randomness involved in the stochastic processes.

For the approaches based on mathematical expansion, usually the second order statistical properties, e.g., the PSD/EPSD or auto-correlation function, are taken as the target probabilistic information such that the stochastic process expressed by random functions in time domain has the second order statistical properties identical to or approximate the target ones. The resulted basic random variables might have, but mostly have no, physical meaning. In this class, among others the Karhunen-Loève expansion method [7,25] and the spectral representation (SR) method [17] were widely adopted in practical applications. However, for the processes with short correlation time usually very large number, say in hundreds, of terms should be retained. Some efforts have been made to reduce the number in the above representations [32,23,20]. Besides, these approaches could only approximate, rather than accurately reproduce, the target statistical properties when the expansion is truncated to retain finite terms.

In the present paper a stochastic harmonic function (SHF) representation is extended to the simulation of non-stationary stochastic processes. It is demonstrated that this new SHF representation could reproduce the target second order statistical properties, e.g., the EPSD, exactly, no matter how many terms are retained. Besides, the distribution of the SHF processes is asymptotically Gaussian. Numerical examples are also addressed, demonstrating the accuracy of the proposed approach.

2. Spectral representations of evolutionary stochastic processes

2.1. Stationary stochastic processes

Let $X(t)$ be a zero-mean, second order stationary processes with the power spectral density function (PSD) $S_X(\omega)$, i.e., $X(t)$ could be represented by the following stochastic function in frequency domain, $Z(\omega)$, such that [29]

$$X(t) = \int_{-\infty}^{\infty} e^{\mathrm{i}\omega t} \mathrm{d}Z(\omega), \tag{1}$$

where $\mathrm{i} = \sqrt{-1}$ is the unit of imaginary number; $Z(\omega)$ is an orthogonal stochastic process with

$$\begin{cases} E[\mathrm{d}Z(\omega)] = 0, \\ E[\mathrm{d}Z^*(\omega)\mathrm{d}Z(\omega')] = E[|\mathrm{d}Z(\omega)|^2]\delta(\omega-\omega') = \mathrm{d}H(\omega) = \delta(\omega-\omega')S_X(\omega)\mathrm{d}\omega, \end{cases} \tag{2}$$

in which $\delta(\cdot)$ is Dirac's delta function; $E[\cdot]$ is the expectation operator; the asterisk "$*$" means the complex conjugate.

Therefore, the auto-correlation function of the process $X(t)$ is given by

$$\begin{aligned} R_X(t,\tau) &= E[X^*(t)X(t+\tau)] \\ &= \int_{-\infty}^{\infty} e^{-\mathrm{i}\omega t} e^{\mathrm{i}\omega'(t+\tau)} E[\mathrm{d}Z^*(\omega)\mathrm{d}Z(\omega')] \\ &= \int_{-\infty}^{\infty} e^{\mathrm{i}\omega\tau} E[|\mathrm{d}Z(\omega)|^2] \\ &= \int_{-\infty}^{\infty} e^{\mathrm{i}\omega\tau} S_X(\omega)\mathrm{d}\omega, \end{aligned} \tag{3}$$

Eq. (3) means that the auto-correlation function of a stationary process is the inverse Fourier transform of its PSD. Conversely, the PSD is the Fourier transform of its auto-correlation function, i.e.,