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The effect of copulas on time-variant reliability involving time-continuous stochastic processes

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ABSTRACT

In structural reliability the dependence structure between random variables is almost exclusively modeled by Gauss (normal or Gaussian) copula; however, this implicit assumption is typically not corroborated. This paper is focusing on time-variant reliability problems with continuous stochastic processes, which are collection of dependent random variables and to our knowledge are not modeled by other than Gauss copula in structural reliability. Therefore, the aim of this contribution is to qualitatively and quantitatively analyze the impact of this copula assumption on failure probability. Three illustrative examples are studied considering bivariate Gauss, *t*, rotated Clayton, Gumbel, and rotated Gumbel copulas. Time-variant actions are modeled as stationary, ergodic, continuous stochastic processes, and the PHI2 method is adopted for the analyses. The calculations show that the copula function has significant effect on failure probability. In the studied examples, application of Gauss copula can four times underestimate or even 10 times overestimate failure probabilities obtained by other copulas. For normal structures agreement on copula type is recommended, while for safety critical ones inference of copula type from observations is advocated. If data are scare, multiple copula functions and model averaging could be used to explore this uncertainty.

1. Introduction

1.1. Problem statement

Problems in structural reliability require the integration of multidimensional joint probability density functions. Due to scarcity of available information, the joint density function is overwhelmingly given by its marginals and by a single parameter dependence measure between any two random variables. The typical dependence measure is the Pearson correlation coefficient; however, this single parameter does not uniquely determine the joint density function. This means that infinitely many joint density functions can be found to the prescribed marginals and correlation coefficient. This indeterminacy is mostly resolved by using Gauss copulas; however, this assumption is typically not verified. To our knowledge, the impact of this copula assumption on time-variant structural reliability problems has not yet been studied. Thus the aim of contribution is to fill this gap by examining the effect of copula function type on out-crossing rate and on failure probability. This study is the extension of an earlier conference paper of the authors [1].

1.2. Motivation and overview of previous works

In structural reliability until recently it was a prevalent, implicit assumption that the dependence structure follows Gauss copula. However, some studies imply that this assumption is not justified by data, and can lead to severe under- or overestimation of failure probability. [2] showed that in geotechnical problems the function adopted to describe the dependence structure between cohesion and friction angle has significant effect on the failure probability. They indicated that the measurements are best described by Plackett copula, and the copula type might lead to an order of magnitude difference in the failure probability for slope stability. Further geotechnical engineering related studies arrived to similar conclusions [3,4]. A study on the impact of copulas on two-component parallel systems concluded that the tail dependence of copulas has significant effect on system reliability [5]. These results are particularly important for the current study, since the herein adopted PHI2 method formulates the time-variant

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reliability problem as a parallel system. Yet, there are important differences due to the nature of time-variant reliability problems. In this paper: (i) system reliability calculation is only one step in the analysis; (ii) the components have substantially different probabilities (failure and survival too); and (iii) the component's failure probability is typically much lower than that of considered in the mentioned study. Other structural engineering applications of copulas involve multi-variate seismic demand modeling [6,7], estimating wind effect on buildings by synthesizing aerodynamic and climate data [8], and piping erosion failure estimation in flood defenses [9].

Copulas are also widespread in many other fields, they are applied for instance for correlated stress-strength models [10], for risk assessment of dam overtopping considering bivariate distribution of flood peak and volume [11], for describing joint distribution of drought indices [12], for economic time series [13], and for modeling multivariate random variables in financial risk [14,15]. All referred studies draw attention to the importance of dependence structure modeling. Moreover, some show the grave consequences of Gauss copula assumption.

The referred works illustrate that the Gauss copula assumption is prevalent and in many cases severely biased. This is the main motivation of this paper to qualitatively and quantitatively investigate the effect of copulas on time-variant reliability problems. We restrict our attention to stationary, ergodic stochastic processes. These are of considerable interest and actively researched and applied to engineering challenges [16–23]. [24] analyzed the effect of parametric uncertainty on time-variant reliability problems; however they did not consider probabilistic model uncertainty, which also stems from limited data, thus belongs to statistical uncertainties. This probabilistic model uncertainty in respect of dependence structure is the focus of this paper.

The rest of the paper is structured as follows: in Section 2, the concept of stochastic processes and copula functions is briefly introduced and the adopted types are described. Thereafter, the PHI2 method and applied numerical techniques are outlined. Then, three illustrative examples are presented: (i) parametric analysis of a minimal problem with a single stochastic process; (ii) a simply supported beam with degrading resistance, which is also subjected to stochastic loading; and (iii) a snow related stochastic problem, where copulas are fitted to observations.

2. Stochastic processes

A stochastic process is a sequence of random variables, which are typically dependent and the strength of this dependence is the function of their distance. Here only time-continuous stochastic processes are considered where the distance is measured in time. Furthermore, the focus is restricted to stationary and ergodic processes. Under these assumptions, the stochastic process is fully characterized by its marginal distribution, autocorrelation function, and dependence (copula) function. The former describes the distribution of intensity in an arbitrary point in time. The autocorrelation function specifies the strength of dependence between random variables separated by a given time interval (Δt). This function applies only to the parameter that measures dependence, and the copula function is required to fully specify the dependence structure.

2.1. Autocorrelation function

Gaussian type autocorrelation function is the common choice in literature. Herein, it is accompanied by Cauchy function to explore the effect of changing the function type (Fig. 1). These two are applied in this work and defined as:

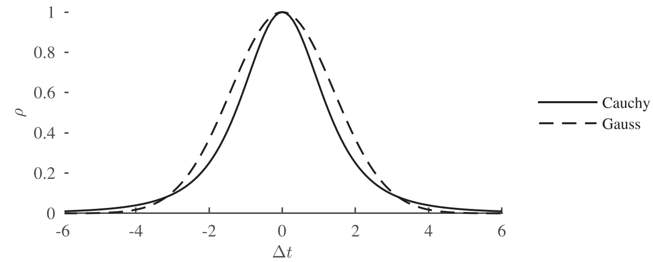

Fig. 1. Illustration of the considered autocorrelation functions for a particular correlation length.

$$\rho_{\mathrm{Gauss}}(\Delta t) = \exp\left(-\left(\frac{\Delta t}{\tau_{\mathrm{F}}}\right)^2\right) \quad \text{and} \tag{1}$$

$$\rho_{\mathrm{Cauchy}}(\Delta t) = \left(1 + \left(\frac{\Delta t}{\tau_{\mathrm{F}}}\right)^2\right)^{-2} \tag{2}$$

where:

τ_{F} correlation length;
ρ Pearson correlation coefficient.

Albeit the latter can express only linear dependence, it is applied here due to its ubiquity in the literature. Kendall's tau and Spearman's rho are more general single parameter measures. For instance, they are sensitive to any monotonic dependence, thus better candidates for dependence measure.

To allow direct comparison with published results and to utilize the more general Kendall's tau measure, the functions given in Eqs. (1 and (2) are kept and the following transformation is used:

$$\tau(\Delta t) = \frac{2}{\pi} \cdot \mathrm{asin}(\rho(\Delta t)). \tag{3}$$

The equivalence of copulas is then set in terms of Kendall's tau values. Formula (3) transforms between Pearson's rho (ρ) and Kendal's tau (τ) for Gauss and t copulas, thus the connection to literature is established while allowing for easy extension to other copulas.

2.2. Copula based dependence structure

Copula is a multivariate probability distribution function with uniformly distributed marginals. It can be used to model the dependence between random variables with arbitrary marginal distributions [25]. In this paper, we deal mostly with bivariate distributions – the formulas are given for that case – though they can be extended to higher dimensions [25]. The copula function is expressed as:

$$C(u_1, u_2; \theta) = P\{U_1 \leqslant u_1, U_2 \leqslant u_2\} \tag{4}$$

where:

$C(.)$ copula (distribution) function;
θ copula parameter;
U_1, U_2 uniformly distributed random variables.

The copula function uniquely describes the dependence structure and it is independent of the marginals if those are continuous. Sklar's theorem establishes the connection between marginals, copula, and joint distribution [26,27]:

$$F_{X1X2} = C(F_{X1}, F_{X2}; \theta) \tag{5}$$