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Asymptotic Results for a Markov-Modulated Risk Process with Stochastic Investment

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Abstract

In this paper we consider a Markov-modulated risk model, where the premium rates, claim frequency and the distribution of the claim sizes vary depending on the state of an external Markov chain. The free reserves of the insurer are invested in a risky asset whose prices are modelled by a geometric Brownian motion, with parameters that are also influenced according to the external Markov process. A system of integro-differential equations for the ruin probabilities and for the expected discounted penalty function is derived. Using Laplace transforms and regular variation theory, we investigate the asymptotic behaviour of both quantities for the case of light or heavy tailed claim size distributions. Specifically, within this set up (where we lose the strong Markov property of the risk process), we show that the ruin probabilities decrease asymptotically as a power function in the case of the light tailed claims, whilst for the heavy tails we show that the probabilities of ruin decay either like a power function, depending on the parameters of the investment, or behave asymptotically like the tails of the claim size distributions.

Keywords: Markov-modulated risk process, Investment, Integro-differential equation system, Ruin probabilities, Expected discounted penalty function, Regular variation, Frobenius method for systems.

1 Introduction

The investigation of insurance risk models with stochastic return on investments has attracted a lot of attention in recent years. Stimulated by the paper of Paulsen (1993) and

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