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Convex integral functionals of regular processes

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Abstract

This article gives dual representations for convex integral functionals on the linear space of regular processes. This space turns out to be a Banach space containing many more familiar classes of stochastic processes and its dual can be identified with the space of optional random measures with essentially bounded variation. Combined with classical Banach space techniques, our results allow for a systematic treatment of stochastic optimization problems over BV processes and, in particular, yields a maximum principle for a general class of singular stochastic control problems.

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1. Introduction

This article studies convex integral functionals of the form

$$EI_h(v) = E \int_0^T h_t(v_t) d\mu_t$$

defined on the linear space \mathcal{R}^1 of regular processes in a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P)$. Here μ is a positive optional measure on $[0, T]$ and h is a convex normal integrand on $\Omega \times [0, T] \times \mathbb{R}^d$. An optional cadlag process v of class (D) is *regular* if $E v_{\tau^\nu} \rightarrow E v_\tau$ for every increasing sequence of stopping times τ^ν converging to a finite stopping time τ or equivalently (see [9, Remark 50d]), if the predictable projection and the left limit of v coincide.

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1 Regular processes is quite a large family of stochastic processes containing e.g. continuous
 2 adapted processes, Levy processes and Feller processes as long as they are of class (D) . A
 3 semimartingale is regular if and only if it is of class (D) and the predictable BV part of its
 4 Doob–Meyer decomposition is continuous.

5 Inspection of [3] reveals that \mathcal{R}^1 is a Banach space under a suitable norm and its dual may be
 6 identified with the space \mathcal{M}^∞ of optional random measures with essentially bounded variation.
 7 Our main result characterizes the corresponding conjugate and subdifferential of $E I_h$ under
 8 suitable conditions on the integrand h . Our main result applies, more generally, to functionals
 9 of the form $E I_h + \delta_{\mathcal{R}^1(D)}$, where $\mathcal{R}^1(D)$ denotes the convex set of regular processes that, outside
 10 an evanescent set, take values in $D_t(\omega) := \text{cl dom } h_t(\cdot, \omega)$. Here, as usual, $\delta_{\mathcal{R}^1(D)}$ is the *indicator*
 11 *function* of $\mathcal{R}^1(D)$ taking the value 0 on $\mathcal{R}^1(D)$ and $+\infty$ outside of $\mathcal{R}^1(D)$.

12 Our main result allows for functional analytic treatment of various stochastic optimization
 13 problems where one minimizes an integral functional over the space of BV-processes. Our
 14 original motivation came from mathematical finance where BV-processes arise naturally as
 15 trading strategies in the presence of transaction costs. In this paper, we give an application to
 16 singular stochastic control by deriving a dual problem and a maximum principle for a fairly
 17 general class of singular control problems that extends and unifies singular control models of
 18 e.g. [2,4,11,18]. Applications to mathematical finance will be given in a separate article.

19 The main result of this paper combines convex analysis with the general theory of stochastic
 20 processes. More precisely, we employ the duality theory of integral functionals on the space
 21 of continuous functions developed by [28] combined with Bismut’s characterization of regular
 22 processes as optional projections of continuous stochastic processes; see [3]. Our main result
 23 states that if the conjugate h^* of h is the optional projection of a convex normal integrand that
 24 allows for Rockafellar’s dual representation of I_h scenario wise, then under mild integrability
 25 conditions, the dual representation of $E I_h + \delta_{\mathcal{R}^1(D)}$ is given simply as the expectation of that of
 26 $I_h + \delta_{C(D)}$, where $C(D)$ denotes the continuous selections of D . The proof is more involved than
 27 the classical results on integral functionals on decomposable spaces or on spaces of continuous
 28 functions. To treat the space of regular processes, techniques from both cases need to be
 29 combined in a nontrivial way. Our proof is based on recent results on optional projections of
 30 normal integrands from [17] and conjugate results for continuous functions from [22].

31 2. Integral functionals and duality

32 This section collects some basic facts about integral functionals defined on the product of a
 33 measurable space (Ξ, \mathcal{A}) and a Suslin locally convex vector space U . In the applications below,
 34 Ξ is either Ω , $[0, T]$ or $\Omega \times [0, T]$. Recall that a Hausdorff topological space is *Suslin* if it
 35 is a continuous image of a complete separable metric space. We will also assume that U is
 36 a countable union of Borel sets that are Polish spaces in their relative topology. Examples of
 37 such spaces include separable Banach spaces as well as their topological duals when equipped
 38 with the weak*-topology. Indeed, such dual spaces are Suslin [32, Proposition A.9] and their
 39 closed unit balls are metrizable in the weak*-topology by [10, Theorem V.5.1], compact by the
 40 Banach–Alaoglu theorem, and thus separable by [10, Theorem I.6.25].

41 A set-valued mapping $S : \Xi \rightrightarrows U$ is *measurable* if the inverse image $S^{-1}(O) := \{\xi \in$
 42 $\Xi \mid S(\xi) \cap O \neq \emptyset\}$ of every open $O \subseteq U$ is in \mathcal{A} . An extended real-valued function
 43 $f : U \times \Xi \rightarrow \overline{\mathbb{R}}$ is said to be a *normal integrand* if the *epigraphical mapping*

$$44 \quad \xi \mapsto \text{epi } f(\cdot, \xi) := \{(u, \alpha) \in U \times \mathbb{R} \mid f(u, \xi) \leq \alpha\}$$

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