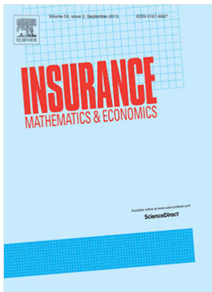
Accepted Manuscript



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PII: S0167-6687(17)30279-2
DOI: https://doi.org/10.1016/j.insmatheco.2018.01.003
Reference: INSUMA 2439

To appear in: *Insurance: Mathematics and Economics*

Received date : June 2017
Revised date : January 2018
Accepted date : 5 January 2018

Please cite this article as: Lin F., Peng L., Xie J., Yang J., Stochastic distortion and its transformed copula. *Insurance: Mathematics and Economics* (2018), https://doi.org/10.1016/j.insmatheco.2018.01.003

Stochastic distortion and its transformed copula

Feng Lin[*], Liang Peng[†], Jiehua Xie[‡] and Jingping Yang[§]

January 2018

Abstract

Motivated by wide applications of distortion functions and copulas in insurance and finance, this paper generalizes the notion of a deterministic distortion function to a stochastic distortion, i.e., a random process, and employs the defined stochastic distortion to construct a so-called transformed copula by stochastic distortions. One method for constructing stochastic distortions is provided with a focus on using time-changed processes. After giving some families of the transformed copulas by stochastic distortions, a particular class of transformed copulas is applied to a portfolio credit risk model, where a numeric study shows the advantage of using the transformed copulas over the conventional Gaussian copula and the double t copula in terms of the fitting accuracy and the ability of catching tail dependence.

Keywords: Stochastic distortion, Transformed copula by stochastic distortions, Time-changed process, Portfolio credit risk model.

1 Introduction

A function $D(u), u \in [0,1]$ is called a distortion function if it is non-decreasing with $D(0) = 0$ and $D(1) = 1$. A distortion function is also called a weighting function or a probability distortion in economic and behavioral studies, and it is one of several key elements of Kahneman and Tversky's Nobel-prize-winning theory, i.e., Prospect Theory and Cumulative Prospect Theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). In Prospect Theory and Cumulative Prospect Theory, an inverse s-shaped distortion function is applied to enlarge the low probability and dismiss the large probability so as to reflect the human subjective probability. Distortion functions were also applied in an experimental design on Cumulative Prospect Theory in Harrison and Swarthout (2016). With the advent of Cumulative Prospect Theory, distortion functions have been used in portfolio selection and behavioral related study. For example, Aït-Sahalia and Brandt (2001), Jin and Zhou (2011) and Carassus and Rasonyi (2015) studied the portfolio selection problems under utility framework by combining with distortion functions; Cohen and Jaffray (1988), Bleichrodt and Pinto (2000) and Bruhin et al. (2010) applied probability distortions to study the behavioral risk-taking and decision-making; Trepel et al. (2005), Zhang and Maloney (2012) and Stauffer et al. (2015) employed distortion functions to the study of neuroscience.

There is a long history of applying distortion functions to risk measures. For example, Yaari (1987) formally applied distortion functions in the dual theory of choice under risk; Wang (1996) defined the Wang's premium principle by using distortion functions and introduced so-called distortion risk measures, which covers some well-known risk measures such as Value-at-Risk and expected shortfall. The distortion

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