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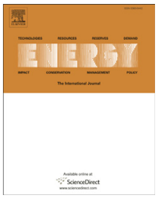

Modeling oil production and its peak by means of a stochastic diffusion process based on the Hubbert curve

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ABSTRACT

The present paper introduces a new diffusion process for the purpose of modeling a Hubbert-type behavior pattern. The main features of the process will be analyzed and the maximum likelihood estimation of the parameters will be carried out through discrete sampling. To this end, a complex system of equations must be solved through numerical procedures, requiring the search for an appropriate initial solution. To this end, we propose three search procedures.

The estimation of peak time (and consequently peak value) is approached from two perspectives: one, by obtaining the maximum likelihood point estimation of the two values (both of them can be expressed as parametric functions); the other, by formulating the peak-time as a first-passage-time problem. Finally, in order to validate the methodology developed, we carry out some studies based on simulated data, and then we consider some real-world applications to crude oil production data for Norway and Kazakhstan.

1. Introduction

Nowadays, global economic growth and political stability are directly associated with the progress of oil production. Given that petroleum is a non-renewable and finite resource, the problem of forecasting future oil production has become of great interest in the last few decades, together with the problem of predicting the time instant at which oil production will peak, and after which production will start to decline.

In 1956, M.K. Hubbert [1] was the first researcher to study this question. Concretely, he considered oil production in the US lower 48 states and correctly predicted its peak around 1970. In that paper he did not provide a functional form for his prediction, presenting instead some graphic methods related to a bell-shaped curve in which the area under the curve was equal to the estimate of the total amount of oil. Later [2], he established that cumulative oil production follows a logistic curve, and thus the yearly production fits its first derivative (called Hubbert's curve since then).

Since the introduction of Hubbert's peak theory, many leading research projects have devoted their efforts at predicting oil production dynamics, particularly regarding peak, peak-time, and resource exhaustion. Some experts, such as Bakhtiari [3] or Deffeyes [4], believe that peak has already been reached, whereas others argue that it will occur soon; concretely, Towler [5] concludes that peak is not likely to occur before 2018, and that even this date may be further extended by rising oil prices and technology developments.

Today, there is an open debate concerning oil depletion. In consequence, several contributions have been made to this field in recent years, proposing analyses and predictions of specific peak-oil dates. Many of these models refer to Hubbert's approach, trying to extend and update it in an attempt to modify and/or complete some aspects of Hubbert's theory.

In this sense, Laherrère showed in Ref. [6] that oil production in several countries (among others France and the UK) cannot be represented by a single cycle as the classic Hubbert's theory establishes. For this reason, he introduced a model characterized by several cycles, adopting an approach that was called since then *multiple-Hubbert modeling*. This approach has been later extended in Ref. [7] by Maggio and Cacciola and applied to modeling oil production. For example, Nashawi et al. [8] considered world oil production, whereas Saraiva et al. dealt with the Brazilian case [9].

Another aspect that has received special attention is the

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symmetry of the Hubbert curve around the peak, which means that the increase and decrease rates of the production (before and after the peak is achieved) are the same. Brandt considered in Ref. [10] several symmetric and asymmetric linear and exponential models, and compared them with the Hubbert curve. Hallock et al. [11], introduced a modified version of the Hubbert curve that peaks at 60% of oil production instead of the typical 50% value exhibited by the Hubbert curve, which implies an asymmetric shape of production and a steeper rate of decline than increase.

In addition, the economic implications of oil production are obvious. Hubbert's method assumes that oil production is only time-dependent and does not take into account the effects of possible technological and/or economic factors. In order to include these factors in such studies, several modifications have been proposed. For example, Reynolds included prices and costs in Hubbert's model [12]; Brecha [13] considered a hybrid model combining the usual logistic curve with extraction costs projected from existing data; and in Ref. [14], Holland considered models in which the production increases, among other factors, with demand, new technologies, and reserve additions. Also, econometric models have been considered, such as that of Kemp and Kasim [15], based on a system of simultaneous equations. Guseo et al. [16] relied on a generalized Bass model to treat global oil growth as a natural diffusion process linked with key exogenous variables like price, technology, and strategic interventions. This model employed autoregressive correlation techniques to dissect historical production time-series data into elements of diffusion, long-memory interventions, and stochastic white-noise components.

Finally, and as regards probabilistic models, Bertrand suggested in Ref. [17] a model to predict oil production based on field size (modeled by a Pareto distribution), which took into account the process of launching production (modeled by a gamma distribution).

As can be seen, the debate on oil depletion has become broader, with several contributions that propose analyses and predictions of the date of peak oil. Many of these models refer to Hubbert's approach, trying to extend and update it. We must note that, although the peak theory pioneered by Hubbert was implemented in the oil production context, its related depletion analysis can be applied to other non-renewable resources such as natural gas (Soldo [18]), phosphorus (Déry and Anderson [19]), and lithium (Vikström et al. [20]).

Most of the models mentioned above do not consider oil production as a random variable that evolves over time. Nevertheless, growth phenomena such as oil production are not static but dynamic, and they show an evolution in time. They usually are rather complex, requiring the specification of several factors which are not always known or quantifiable. These two aspects can be dealt with through the use of stochastic processes, and among them through diffusion processes. In the context of curves related to growth phenomena, such processes arise when a random fluctuation is introduced in the differential equation whose solution is the curve under study, thus introducing stochastic differential equations (SDE).

This methodology has generated a wide range of works in many applied scientific areas. In the context of energy studies, Giovanis and Skiadas [21] used a logistic diffusion process in order to study energy consumption in Greece and the United States. However, other diffusion processes rather than logistic have also been applied in this context. For example, Gutiérrez et al. [22,23] considered a gamma and a Gompertz diffusion process, respectively, for modeling the trend of the total stock of private oil-driven cars and electricity consumption in Morocco. More recently, Nafidi et al. [24] considered a gamma diffusion process with exogenous factors with the goal of modeling and predicting electricity consumption in Spain.

Nevertheless, diffusion processes have not been widely used for modeling oil production. In this paper we consider a stochastic diffusion model related to the Hubbert curve. This process models the evolution of oil production over time, and provides information regarding production peak. As a matter of fact, the process verifies that its mean function and conditioned mean function (conditioned on a value at a previous time instant) are Hubbert curves. This indicates that the process can be considered as a valid tool for studying data showing a Hubbert-type behavior, such as oil production data. In particular, these mean functions allow us to adjust the observed data over time and to predict future values of oil production. On the other hand, the fact that these mean functions are Hubbert curves enables the researcher to study peak and peak-time. We have to note that mean functions, peak and peak-time can be expressed as parametric functions, so in order to estimate them we must first estimate the parameters of the process. In addition, using a diffusion process to model the time evolution of oil production allows us to address the study of peak-time from a stochastic perspective, and more specifically as the time variable at which production achieves its maximum. This question can be formulated as a first-passage-time (FPT) problem.

The paper is structured as follows: Section 2 describes how the Hubbert process is obtained, and presents its mean characteristics. An inferential study of the parameters of the process is dealt with in Section 3. The inferential study is carried out on the basis of discrete sampling and includes numerical aspects related to the search of initial solutions to be taken into account for solving the likelihood equations. In Section 4, we consider the first-passage-time problem related to the time at which the peak of oil production is achieved. In Section 5, and with the considerations made in the previous two sections as a starting point, procedures to estimate peak and peak-time are proposed. In order to validate the methodology described, in Section 6 a simulation study is carried out. Finally, in Section 7, we present two applications to real data by considering crude oil production data from Norway and Kazakhstan. These examples show the possibilities that the process introduced presents for modeling oil production and how it can help answer the question of when peak production can be expected.

2. The model

As noted in the previous section, our aim is to introduce a diffusion process related to the Hubbert curve, which allows us to model oil production. This curve can be obtained from the derivative of the logistic one. For this reason, the starting point will be a general expression of the logistic curve,

$$f(t) = \frac{a}{1+be^{-ct}}, \quad t\in\mathbb{R}; a,b,c>0.$$

Noting $\alpha = e^{-c}$, $\eta = 1/b$ and $k = a\eta$, this expression becomes

$$f(t) = \frac{k}{\eta+\alpha^t}, \quad t\in\mathbb{R}; \eta,k>0, 0<\alpha<1, \tag{1}$$

from where, after deriving with respect to t and imposing that $f'(t_0) = x_0$ (t_0 represents the initial observation time), we obtain a general expression for the Hubbert curve:

$$x(t) = f'(t) = x_0\left(\frac{\eta+\alpha^{t_0}}{\eta+\alpha^t}\right)^2 \alpha^{t-t_0}, t\in\mathbb{R}; \eta>0, 0<\alpha<1. \tag{2}$$

Note that, in our context, $x(t)$ represents oil production as a function of time (t, usually expressed in years).

This curve verifies the following properties: