Integrably bounded set-valued stochastic integrals

Michał Kisielewicz *, Mariusz Michta *

Faculty of Mathematics Computer Science and Econometrics, University of Zielona Góra, Podgórna 50, 65-246 Zielona Góra, Poland

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ABSTRACT

The paper is devoted to properties of Aumann and Itô set-valued stochastic integrals, defined as some set-valued random variables. In particular the problem of integrable boundedness of the generalized Itô set-valued stochastic integrals is considered. Unfortunately, Itô set-valued stochastic integrals, defined by E.J. Jung and J.H. Kim in the paper [5], are not in general integrably bounded (see [8,15]). Therefore, in the present paper we consider generalized Itô set-valued stochastic integrals (see [10,11]) defined for absolutely summable and countable subsets of the space \( L^2(\mathbb{R}^+ \times \Omega, \Sigma_F, \mathbb{R}^{d \times m}) \) of all square integrable \( \mathcal{IF} \)-nonanticipative matrix-valued stochastic processes. Such integrals are integrably bounded and possess properties needed in the theory of set-valued stochastic equations.

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1. Introduction

The paper deals with properties of Aumann and Itô set-valued stochastic integrals, defined as some set-valued random variables. Initial studies on Itô set-valued stochastic integrals, defined as subsets of the spaces \( L^2(\Omega, \mathbb{R}^n) \) and \( L^2(\Omega, \mathcal{H}) \), have been considered by F. Hiai and M. Kisielewicz (see [2,6,7]), where \( \mathcal{H} \) is a Hilbert space. Unfortunately, such defined integrals do not admit their representations by set-valued random variables with values in \( \mathbb{R}^n \) and \( \mathcal{H} \), because they are not decomposable subset of \( L^2(\Omega, \mathbb{R}^n) \) and \( L^2(\Omega, \mathcal{H}) \), respectively. J. Jung and J.H. Kim in [5] have defined the Itô set-valued stochastic integral as a set-valued random variable determined by a closed and decomposable hull of the set-valued stochastic functional integral defined in [6]. Unfortunately, such integrals are not in the general case (see [8,15]) integrably bounded. Therefore, in what follows we shall consider generalized Itô set-valued stochastic integrals (see [10,11]) of absolutely summable countable subsets of the space \( L^2(\mathbb{R}^+ \times \Omega, \Sigma_F, \mathbb{R}^{d \times m}) \) of square integrable \( \mathcal{IF} \)-nonanticipative matrix-valued stochastic processes defined on a complete filtered probability space \( \mathcal{P}_F = (\Omega, \mathcal{F}, \mathcal{IF}, P) \). Generalized set-valued stochastic integrals were defined in the paper [11] and some of their properties have been considered in [10]. Let us recall (see [10] and [11]) that

* Corresponding authors.

E-mail addresses: M.Kisielewicz@wmie.uz.zgora.pl (M. Kisielewicz), M.Michta@wmie.uz.zgora.pl (M. Michta).

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for a given $m$-dimensional $\mathbb{IF}$-Brownian motion $B = (B_t)_{t \geq 0}$ defined on a filtered probability space $\mathcal{P}_\mathcal{F}$ and a nonempty subset $\mathcal{G}$ of the space $\mathbb{IL}^2(\Omega \times \mathbb{IF}, \mathbb{IF}^{d \times m})$, a generalized Itô set-valued stochastic integral $\int_0^t \mathcal{G} dB_t$ is understood as an $\mathcal{F}_t$-measurable set-valued random variable with values in the $d$-dimensional Euclidean space $\mathbb{IR}^d$ and subtrajectory integrals $S_{\mathcal{F}_t}(\int_0^t \mathcal{G} dB_t)$ equal to $\delta_{\mathcal{G}}(\mathcal{J}_t(\mathcal{G}))$. By $\mathcal{J}_t$ we denote the Itô isometry with values in the space $\mathbb{IL}^2(\Omega, \mathcal{F}_t, \mathbb{IF}^d)$ defined for fixed $t \geq 0$ and every $g \in \mathbb{IL}^2(\Omega \times \mathbb{IF}, \mathbb{IF}^{d \times m})$ by setting $\mathcal{J}_t(g) = \int_0^t g_t dB_t$. Subtrajectory integrals $S_{\mathcal{F}_t}(\int_0^t \mathcal{G} dB_t)$ of $\int_0^t \mathcal{G} dB_t$ are defined as a set of all $\mathcal{F}_t$-measurable and square integrable selectors of $\int_0^t \mathcal{G} dB_t$. It will be also denoted by $S_t(\int_0^t \mathcal{G} dB_t)$. In particular, if $\mathcal{G}$ is a nonempty decomposable subset of $\mathbb{IL}^2(\Omega \times \mathbb{IF}, \mathbb{IF}^{d \times m})$ then $\int_0^t \mathcal{G} dB_t = \int_0^t \mathcal{G} dB_t$, where $\mathcal{G} = (G_t)_{t \geq 0}$ is an $\mathbb{IF}$-nonanticipative set-valued process such that

$$S_{\mathcal{F}_t}(\mathcal{G}) = c(\mathcal{G}(\mathcal{G})),$$

where $S_{\mathcal{F}_t}(\mathcal{G}) = \{g \in \mathbb{IL}^2(\Omega \times \mathbb{IF}, \mathbb{IF}^{d \times m}) : g_t(\omega) \in G_t(\omega) \text{ for a.e. } (t, \omega) \in \Omega \times \mathbb{IF} \}$. In a similar way the Aumann set-valued stochastic integrals can be defined (see e.g. [9], p. 114 and [16]).

Namely, for a given $\mathbb{IF}$-nonanticipative set-valued stochastic process $F : \Omega \times \mathbb{IF} \rightarrow \mathcal{C}(\mathbb{IF}^d)$ such that $S_{\mathcal{F}_t}(\mathcal{F} \neq 0$, the Aumann set-valued stochastic integral $\int_0^t F_t dB_t$ is defined as the set-valued random variable such that $S_t(\int_0^t F_t dB_t) = \delta_{\mathcal{G}}(\mathcal{J}_t(\mathcal{F})), \text{ where } \mathcal{J}_t(\mathcal{F}) \text{ denotes for fixed } t \geq 0 \text{ the mapping with values in the space } \mathbb{IL}^2(\Omega, \mathcal{F}_t, \mathbb{IF}^d)$ defined for every $f \in \mathbb{IL}^2(\Omega \times \mathbb{IF}, \mathbb{IF}^d)$ by setting $\mathcal{J}_t(f) = \int_0^t f_t dB_t$.

Apart from the above defined Aumann set-valued stochastic integral $\int_0^t F_t dB_t$ one can define (see e.g. [9, 12] and [13]) an $(\mathcal{A})$-set-valued stochastic integral (A) $\int_0^t F_t dB_t$ by setting ((A) $\int_0^t F_t dB_t(\omega) = \int_0^t F_t(\omega, t) dB_t$ for fixed $t \geq 0$ and a.e. $\omega \in \Omega$, where $\int_0^t F_t(\omega, t) dB_t$ denotes the parametrized Aumann integral, i.e. for a.e. fixed $\omega \in \Omega$ the Aumann integral of $F_t(\cdot, \omega)$. It can be verified (see [9], Lemma 3.1 of Chap. 3 and also [12, 13]) that (A) $\int_0^t F_t dB_t$ is an $\mathcal{F}_t$-measurable convex, compact valued set-valued random variable. But for a.e. $\omega \in \Omega$ one has $(\mathcal{A}) \int_0^t F_t dB_t(\omega) = \{u(\omega) : u \in J_t(\mathcal{S}_{\mathcal{F}_t}(\mathcal{F})))) \subset \{u(\omega) : u \in \delta_{\mathcal{G}}(\mathcal{J}_t(\mathcal{F})), \text{ by } (\int_0^t F_t dB_t(\omega)\text{ denotes the parametrized Aumann integral, i.e. for a.e. fixed } \omega \in \Omega \text{ the Aumann integral of } F_t(\cdot, \omega). \text{ It can be verified (see }[9], \text{ Corollary 3.1 of Chap. 3) that for every measurable, convex-valued and integrably bounded set-valued process } F : \Omega \times \mathbb{IF} \rightarrow \mathcal{C}(\mathbb{IF}^d) \text{ one has } \mathcal{J}_t(F_t dB_t(\omega) = (\int_0^t F_t dB_t(\omega)) \text{ for a.e. } \omega \in \Omega \text{ and } t \geq 0. \text{ For other properties of } (\mathcal{A}) \int_0^t F_t dB_t \text{ we refer the reader to }[9, 12] \text{ and }[13].$

Let us assume that $\mathcal{G} = \{g^n : n \geq 1\} \subset \mathbb{IL}^2(\Omega \times \mathbb{IF}, \mathbb{IF}^{d \times m})$ is absolutely summable, i.e.

$$\sum_{n=1}^{\infty} \|g^n\|^2 < \infty,$$

where $\| \cdot \|$ is a norm of $\mathbb{IL}^2(\Omega \times \mathbb{IF}, \mathbb{IF}^{d \times m})$. Then a generalized set-valued integral $\int_0^t \mathcal{G} dB_t$ can be defined for every $t \geq 0$ as a set-valued random variable $\mathcal{H}_t : \Omega \rightarrow \mathcal{C}(\mathbb{IF}^d)$ of the form $\mathcal{H}_t(\omega) = c(\int_0^t g^n dB_t(\omega) : n \geq 1)$ for every $\omega \in \Omega$. Indeed, it is clear, that for every $t \geq 0$ a sequence $\{\int_0^t g^n dB_t \}_{n=1}^{\infty}$ is the Cauchy representation of a set-valued random variable $\mathcal{H}_t$. Furthermore, we have $\mathcal{H}_t(\omega) = \{u(\omega) : u \in J_t(\mathcal{S}_{\mathcal{F}_t}(\mathcal{F})))) \subset \{u(\omega) : u \in \delta_{\mathcal{G}}(\mathcal{J}_t(\mathcal{F})), \text{ by } (\int_0^t F_t dB_t(\omega)\text{ denotes the parametrized Aumann integral, i.e. for a.e. fixed } \omega \in \Omega \text{ the Aumann integral of } F_t(\cdot, \omega). \text{ It can be verified (see }[9], \text{ Corollary 3.1 of Chap. 3) that for every measurable, convex-valued and integrably bounded set-valued process } F : \Omega \times \mathbb{IF} \rightarrow \mathcal{C}(\mathbb{IF}^d) \text{ one has } \mathcal{J}_t(F_t dB_t(\omega) = (\int_0^t F_t dB_t(\omega)) \text{ for a.e. } \omega \in \Omega \text{ and } t \geq 0. \text{ For other properties of } (\mathcal{A}) \int_0^t F_t dB_t \text{ we refer the reader to }[9, 12] \text{ and }[13].$

Let us also note that for such a set $\mathcal{G} = \{g^n : n \geq 1\}$, a stochastic process $(\sum_{n=1}^{\infty} |\int_0^t g^n dB_t|^2)_{t \geq 0}$ is a positive submartingale. Firstly, let us observe that such a set $\mathcal{G}$ is bounded in $\mathbb{IL}^2(\Omega \times \mathbb{IF}, \mathbb{IF}^{d \times m})$, because

$$\sup_{n \geq 1} \|g^n\|^2 \leq \sum_{n=1}^{\infty} \|g^n\|^2 < \infty.$$
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