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Patterns of a spatial exploration under time evolution of the attractiveness: Persistent nodes, degree distribution, and spectral properties

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HIGHLIGHTS

- A two dimensional spatial exploration by an agent under Boltzmann-like transition probabilities was studied.
- Spectral properties of generated graph drawn by the random walk produced by the agent were analysed.
- Deviates from the universal laws for the density of eigenvalues are also observed.
- There are stationary persistent nodes when these ones have evolutionary attractiveness.
- Power law behaviour for the persistent nodes is observed for a particular non-Boltzmann transition probability based only on the attractiveness of the nodes.

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ABSTRACT

This work explores the features of a graph generated by agents that hop from one node to another node, where the nodes have evolutionary attractiveness. The jumps are governed by Boltzmann-like transition probabilities that depend both on the euclidean distance between the nodes and on the ratio (β) of the attractiveness between them. It is shown that persistent nodes, i.e., nodes that never been reached by this special random walk are possible in the stationary limit differently from the case where the attractiveness is fixed and equal to one for all nodes ($\beta = 1$). Simultaneously, one also investigates the spectral properties and statistics related to the attractiveness and degree distribution of the evolutionary network. Finally, a study of the crossover between persistent phase and no persistent phase was performed and it was also observed the existence of a special type of transition probability which leads to a power law behaviour for the time evolution of the persistence.

1. Introduction

The idea of preferential attachment of Barabasi and Albert [1,2] has brought a revolution to the study of complex systems mainly by the simplicity of the idea and its wide applicability which goes from web, epidemics, metabolic networks, scientific collaborations, human mobility and so on. Thus, a really wide scope of the extensions of this and many other ideas in network science have been developed.[1]

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[1] http://barabasi.com/networksciencebook/.

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An interesting point in the theory of evolutionary networks is related to the question of how random walks, with properties on the edges or on the nodes, are able to perform this evolution starting from initial condition where only nodes (no edges) exist and are distributed in a two-dimensional surface.

In this context certain known statistical properties of random walks as first time passage [3] and also properties related to the statistical fluctuation in spin systems as persistence [4–6] should be closely linked with properties of the generated graph formed by the path (sequence of the visited edges and nodes) after successive hops of an agent (not simply a walker, since this entity may not only move to spatially near sites in a more general case) which takes into account the dependence on the spatial distribution of the nodes and their peculiarities. Following this idea, the agent should be guided by the euclidean distance between the nodes. Moreover these nodes also should have a kind attractiveness characterized by the frequency which they are visited along the time evolution, remembering in some sense, of preferential attachment method proposed in [1].

Following this direction, the internet is a good example. It can be imagined that a particular individual have some distance in relation to some topics. One has some preference list and navigates in the internet, reading news about science, maybe politics, and other topics. This means that she(he) keeps a short distance in relation to these topics of her(his) preference. On the other hand, she (he) should have high distances, for example, in relation to topics such as sports, religion, social gossips. Thus the access probability to these no preferential topics is initially small. However, when such topics have been accessed, they call attention even of non-usual users increasing the access probabilities and contributing to the formation of the so-called "trending topics".

Looking at the graph properties, it can be highlighted some basic properties as the degree distribution, access distribution but also spectral properties as the density of eigenvalues of matrices related to adjacency matrix of the random walk generated graph. The idea of considering a random walk in a set of random points in a two-dimensional space brings a lot of interesting discussion in Physics and an important question refers to the problem of a particular node not being visited at large times, or translating to a spin system the question change to what the probability that a particular spin does not change its state until time t.

Such concept was deeply studied by many authors, considering dynamics at temperature $T = 0$, known as coarsening dynamics (see [5]). A simple dynamics in this context, for example considers that if a flip of a particular spin state interacting only with its first neighbours, decreases the energy system, its state must be changed otherwise it changes with probability $1/2$. In this case the fraction of spins with unaltered state from the beginning to the time t in one, two, or three-dimensional lattices decays as power law as function of time

$$Pers(t) \sim t^{-\theta}. \tag{1}$$

Such behaviour is exponentially stretched for $T \neq 0$. This concept is known as local persistence. Similarly, this concept has a global version [4]. In this case, it can be shown that for systems at critical temperature, the probability of magnetization (essentially the sum of spin states divided by the system size) does not have changed its sign until time t also decays as a power law given in Eq. (1), but characterized by a different exponent.

Thus, the existence of persistent sites in the graph formed by the random walk deserves some attention, or alternatively, if there is a power law decay for the persistence which means that some nodes are persistent at large times.

However, if the persistence decays exponentially, the question must be changed and in this case it is interesting to predict if the system presents a residual fraction of persistent nodes in the limit:

$$\lim_{t\to\infty} Pers(t) = p_\infty \tag{2}$$

with $p_\infty > 0$. Starting from this idea, let us imagine that a navigation of a particular people in the web can be mimicked as a random walk with some idiosyncrasies. In this particular random walk, one supposes that the agent has some initial distances in relation to the topics, however after a visit to a particular topic, the attractiveness of this site, which was not even the most important, has changed given its visit, and the probability of visit to this topic now, not only depends on the distance but also depends of its attractiveness that initially was the same for all sites.

Hence, by considering that the hops are initially governed only by the distances of the agent in relation to the nodes and that after some time the incidence of the agent on the nodes make them more attractive, the important question here is: what are the properties of the graph formed during the evolution of this peculiar random walk? In other words, it is considered a updating dynamics to evolve the potential of the nodes based on a kind of reinforcement mechanism to change the initial effects of the distance on the transition probabilities among the different pairs of nodes, and the focus is the exploration of nodes over time.

In a more abstract point of view, it can be imagined as mathematical modelling where a agent paints an edge (if this does not exist) when one transits between two nodes and the growing graph obtained from this process (nodes + painted edges) can be studied.

In the literature some authors have explored similar models such for example the step-by-step random walk network model described in [7], and preferential network random walk studied in [8]. All these models have a similar dynamics but different aims. These models are different from our original idea for this current work since they start from a fully connected network of a certain number of nodes m. In [7], a new node is created which will be connected to the certain number of