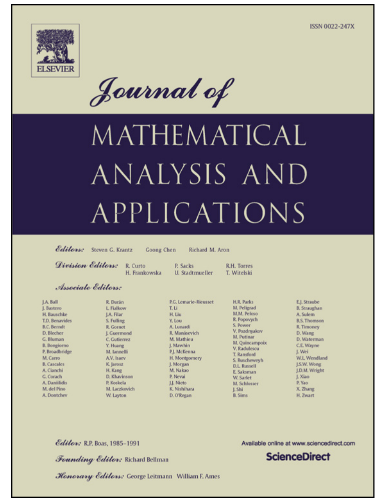

The fundamental analysis method for the blow-up of wave equations in first quadrant

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Abstract

This paper is devoted to prove the sharpness on the lower bound of the life-span of classical solutions to the initial-boundary value problem for one dimensional general quasilinear wave equations in the general case obtained in [10]. What's more, the originality of the paper is the fundamental analysis method of the blow-up and lifespan property, that is to say, instead of differential inequality and the pure mathematical analysis, this is the first attempt to this direction. The highlight of the paper is that we will use the most fundamental D'Alembert's formula of the solutions of wave equation, and the corresponding actual physical meaning, by means of the scaling argument, dilation translation, translation transformation, rotation transformation, we connect our problem with a Goursat problem, and then we can intuitively obtain our blow-up and lifespan result for the problem, this is a new idea in the paper.

Keywords: Wave equations; Blow up; Life-Span; Initial-boundary value problem

1 Introduction and Main Results

In the paper [10], the author has considered the initial-boudary value problem with small initial data and zero boundary data for the following general quasilinear wave equations

$$\begin{cases} u_{tt} - u_{xx} = b(u, Du)u_{xx} + 2a_0(u, Du)u_{tx} + F(u, Du), \quad x > 0, \quad t > 0, \\ t = 0: \ u = \varepsilon\varphi(x), \quad u_t = \varepsilon\psi(x), \quad x > 0, \\ x = 0: \ u = 0, \quad \text{for} \ \ t \geq 0, \end{cases} \tag{1.1}$$

where functions $b(\widetilde{\lambda})$, $a_0(\widetilde{\lambda})$ and $F(\widetilde{\lambda})$ in (1.1) are all sufficiently smooth functions satisfying $b(\widetilde{\lambda}),\ a_0(\widetilde{\lambda}) = O(|\widetilde{\lambda}|^{\alpha})$, $F(\widetilde{\lambda}) = O(|\widetilde{\lambda}|^{1+\alpha})$, and $a(\widetilde{\lambda}) = 1 + b(\widetilde{\lambda}) \geq m_0$, where α is an integer≥ 1, m_0 is a positive constant.

By definition, the life-span $T(\varepsilon) = \sup \tau$ for all $\tau > 0$ such that there exists a classical solution to (1.1) on the time interval $0 \leq t \leq \tau$. The author[10] studied the life-span of classical solutions to (1.1) for all integers $\alpha \geq 1$, and has obtained their results as follows: There exists a small positive number ε_0 such that for any $\varepsilon \in (0, \varepsilon_0]$, the life-span of solutions to problem (1.1) has the following lower bounds in the general case:

$$T(\varepsilon) \geq a\varepsilon^{-\alpha(1+\alpha)/(2+\alpha)}, \tag{1.2}$$

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