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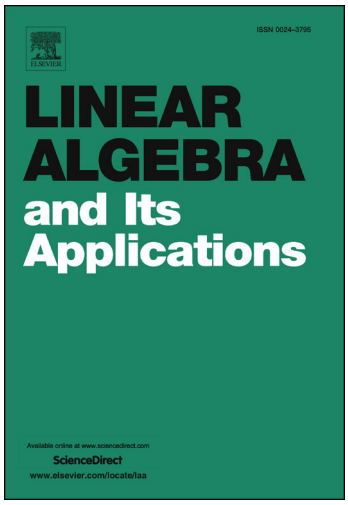

Matrix convex sets without absolute extreme points

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MATRIX CONVEX SETS WITHOUT ABSOLUTE EXTREME POINTS

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Abstract. This article shows the existence of a class of closed bounded matrix convex sets which do not have absolute extreme points. The sets we consider are noncommutative sets, K_X, formed by taking matrix convex combinations of a single tuple X. In the case that X is a tuple of compact operators with no nontrivial finite dimensional reducing subspaces, K_X is a closed bounded matrix convex set with no absolute extreme points.

A central goal in the theory of matrix convexity is to find a natural notion of an extreme point in the dimension free setting which is minimal with respect to spanning. Matrix extreme points are the strongest type of extreme point known to span matrix convex sets; however, they are not necessarily the smallest set which does so. Absolute extreme points, a more restricted type of extreme points that are closely related to Arveson's boundary, enjoy a strong notion of minimality should they span. This result shows that matrix convex sets may fail to be spanned by their absolute extreme points.

1. Introduction

One of the central topics in matrix convexity is the subject of extreme points. In the dimension free setting there are many notions of an extreme point. One class, introduced by Webster and Winkler in [WW99], is the notion of a matrix extreme point. The main result in [WW99] shows that a closed bounded matrix convex set K is spanned by its matrix extreme points, i.e. the closed matrix convex hull of the matrix extreme points of K is equal to K, and is a critical result in the theory of matrix convex sets. However, it is often the case that a proper subset of the matrix extreme points spans K. In fact, a matrix convex combination involving a single matrix extreme point of K can produce a new matrix extreme point [A69, F00, F04]. As of today, it remains unknown if there is a natural notion of extreme points for matrix convex sets which is minimal with respect to spanning.

A more restricted class of extreme point is the notion of an absolute extreme point which was introduced by Kleski [KLS14]. This class of extreme points is closely related to Arveson's notion [A69] of a boundary representation of an operator system [KLS14, EHKM17]. In the

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