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Analytical approach for dynamic instability analysis of functionally graded skew plate under periodic axial compression

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A B S T R A C T

Analytical studies on the dynamic instability analysis of a functionally graded (FG) skew plate subjected to uniform and linearly varying in-plane periodic loadings with four different types of boundary conditions are presented. The total energy functional of the FG skew plate is formulated based on Reddy's third order shear deformation theory (TSDT) and this functional is mapped from the physical domain to computational domain using transformation rule. The boundary characteristics orthonormal polynomials (BCOPs) are generated for different boundary conditions using Gram–Schmidt process, which satisfy the essential boundary conditions of skew plates in the computational domain. The energy functional is converted into a set of ordinary differential equations (Mathieu–Hill equations) using Rayleigh–Ritz method in conjunction with BCOPs. The solution of Mathieu–Hill equations describes the dynamic instability behavior of skew plate. The instability regions are traced using Bolotin method. The effect of skew angles, power-law distributions, span-to-thickness ratios, aspect ratios, boundary conditions and static load factors on the instability region of FG skew plates are presented. The result indicates that the width of instability region become narrow with the increase in skew angle. Moreover, the time history response and corresponding phase plot in the unstable and stable region is studied to identify the instability behavior such as existence of beats, bounded and unbounded response, and effect of forcing amplitude and its frequency on the response.

1. Introduction

Over the decades, functionally graded materials (FGMs) are increasingly used in the defence, aerospace, nuclear reactors and other high-performance structures [1,2]. The FGM has several advantages over FRP composite structures based on material characteristics and thus, they are used more frequently in high-performance structures. The FGM is made up of ceramic and metal having a smooth variance of material properties throughout the thickness. The uses of functional graded material in aerospace/nuclear industries are well known. Therefore, the static and dynamic instability of FG skew plates is essential for design aspects and its performance. Extensive research has been done on the static [3–6] and dynamic instability [7–11] for the structural element made of isotropic and composite type of materials. Recently, Jha et al. [12] provided a critical review on the studies of FG plates.

The FG skew plate starts deflecting outward under in-plane compressive loading because of the extensional-bending coupling like un-symmetric cross-ply laminated skew plates. Ng et al. [13,14] studied the dynamic instability behavior of FG cylindrical thin shells under uniform periodic axial loading. Shen [15] analyzed the bending performance of a simply supported FG rectangular plate under transverse load and pointed out that the FG plate behaves like un-symmetric laminated plate. Since the neutral axis of the FG plate do not pass through the middle plane the plate, it may not possess the bifurcation point, the shift of the neutral axis needs to be considered in the analysis of the FGM plate [16]. The effect of various factors such as gradient index, temperature, aerodynamic loading on the dynamic instability of a FG plate was studied by Lanhe et al. [17] using differential quadrature method.

The nonlinear free flexural vibration of functionally graded skew and rectangular plates was performed by Sundararajan et al. [18] under thermal loadings. Ganapathi and Prakash [19] discussed the influence of the parameters (such as aspect ratio, volume index, skew angle etc.) on the critical buckling load of simply supported FGM skew plate. Pradyumna and Bandyopadhyay [20] studied the dynamic instability of FGM shell under in-plane periodic load and temperature. Torki et al. [21] investigated the flutter behavior of a FGM cylindrical shell subjected to distributed axial forces. Asnafi and Abedi [22] studied the dynamic stability of a FG plates subjected to random lateral loads. Sofiyer and Kuruoglu [23] studied the instability behavior of FG shell structures and sandwich cylindrical shell with FG core under uniform periodic axial loads using the Galerkin's method.

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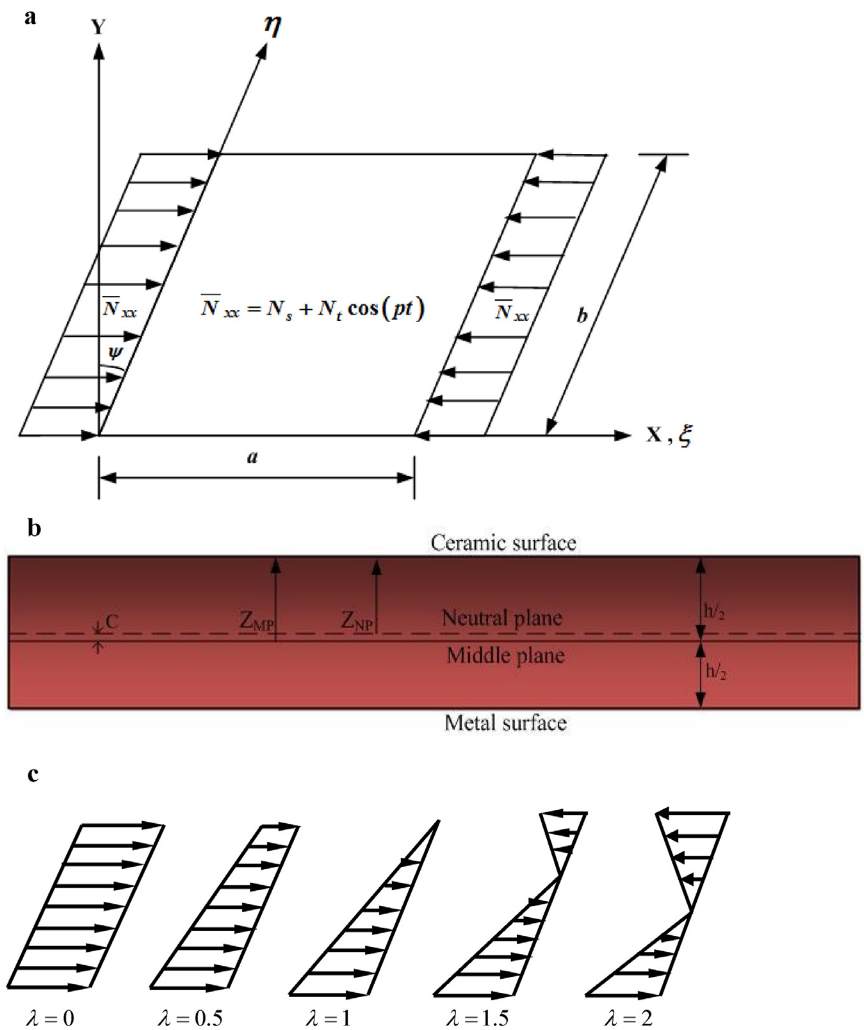

Fig. 1. (a) FG skew plate geometry and uniform periodic axial compression. (b) The position of mid plane and neutral plane for FG skew plate. (c) Different types of linearly varying in-plane edge loadings.

The literature on the dynamic instability of FG skew plate is very limited and required details exploration of design aspects and better performance. In the present study, analytical investigation carried out on the dynamic stability of FG skew plates under uniform and linearly in-plane varying loads using Rayleigh–Ritz Method in conjunction with BCOPs. The total energy functional of the FG skew plate is formulated based on Reddy's TSDT and transformed from physical domain to computational domain using transformation rule. The total energy functional (in computational domain) is then reduced to ordinary differential equations (Mathieu–Hill equations) using Rayleigh–Ritz method in conjunction with BCOPs. The BCOPs are generated by orthogonalization to the boundary characteristic polynomials (BCPs) via Gram–Schmidt process. The BCPs are then taken as a product of two-dimensional independent set of polynomials and the basis functions. The basis functions are developed by taking the product of boundary equation of skew plate, which satisfy the essential boundary conditions. The boundaries of instability regions are plotted using Bolotin method [25]. The effect of different skew angles, various types of linearly varying loadings, static load factors, boundary conditions, span-to-thickness ratios and aspect ratio on the instability regions are studied.

2. Mathematical formulation

The functionally graded (FG) skew plate is made by mixing of ceramic and metal materials in which the bottom surface is metal and the top surface is ceramic. The FG skew plate of length "a", width "b" and thickness "h" under uniform loading shown in Fig. 1(a).

The material distribution along the thickness direction of FG plate is not symmetric about the middle plane. Therefore, the neutral plane may not coincide with the middle plane. Fig. 1b shows the position of the middle plane and the neutral plane of the skew plate at any cross section. In the present case the reference plane taken as a neutral plane for the analysis of functionally graded skew plate using elementary theory to avoid the coupling between the stretching and bending deformation. In order to determine the position of the reference plane from the middle plane of the functionally graded skew plate for which the first moment of modulus of elasticity with respect to reference plane is equated to zero [26,27] as follows,

$$\int_{-h/2}^{h/2} E(z)(z - C)dz = 0 \tag{1}$$

the position of reference plane (neutral plane) from middle plane may be defined as

$$C = \frac{\int_{-h/2}^{h/2} E(z)z\,dz}{\int_{-h/2}^{h/2} E(z)dz} \tag{2}$$

Therefore, distance (z) of the top surface and the bottom surface from the reference plane are $z_t(= \frac{h}{2} - C)$ and $z_b(= -\frac{h}{2} - C)$, respectively. The effective material properties (modulus of elasticity and