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Compressible flows on moving domains: Stabilized methods, weakly enforced essential boundary conditions, sliding interfaces, and application to gas-turbine modeling

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ABSTRACT

A novel stabilized formulation for 3D compressible viscous flows on moving domains is developed. New weak imposition of essential boundary conditions and sliding-interface formulations are also proposed in the context of moving-domain compressible flows. The new formulation is successfully tested on a set of examples spanning a wide range of Reynolds and Mach numbers showing its superior robustness. Experimental validation of the new formulation is also carried out with good success. In addition, the formulation is applied to simulate flow inside a gas turbine stage, illustrating its potential to support design of real engineering systems through high-fidelity aerodynamic analysis.

1. Introduction

The success of finite element methods in solid and structural mechanics, heat conduction, and other areas in 1970s encouraged its development and use to simulate flow problems. Stabilized finite element methods for fluid mechanics were introduced, and the first of them was the streamline upwind/Petrov–Galerkin (SUPG) method [1] for incompressible flows. The key idea of SUPG was to add a residual-based stabilization term to the Galerkin form of the governing equations in order to enhance the stability for higher Reynolds number flows while retaining consistency of the formulation. SUPG was extended to compressible flows using conservation variables in [2–4]. The concept of SUPG was further refined and studied for entropy variables in [5–7], and then generalized to arbitrary variable sets in [8,9]. Over the years, significant progress was made in stabilized methods for compressible flows. The one perhaps most relevant to this paper was combining a new version [10,11] of the compressible-flow SUPG method [2–4] with the Deforming-Spatial-Domain/Stabilized Space–Time (DSD/SST) method [12–14]. The DSD/SST method (now also called the "ST method") was introduced for flow problems with moving boundaries/interfaces, including fluid–structure interaction (FSI). The method resulting from this straightforward mixture of the DSD/SST concept and the compressible-flow SUPG method, which is now called compressible-flow ST SUPG method, was first tested in [15]. This was followed by computations for air intake of a jet engine with adjustable spool at supersonic speeds [16], aerodynamics of two high-speed trains in a tunnel [14], liquid propellant guns [17,18], and compressible-flow FSI [19,20]. Other progress included large-scale parallel computations [16,21–24], unified formulations of incompressible and compressible flows [8,25], and the development of stabilization parameters [10,11,26–32].

It was observed early on that when stabilized methods were applied to compressible flow analysis, oscillations occurred in the vicinity of shocks and other sharp solution features. Hughes et al. [33,34] proposed a class of shock- or discontinuity-capturing methods that provide additional dissipation by adding mesh- and solution-dependent artificial viscosity terms to a stabilized formulation. These viscosities are often residual-based, and thus preserve consistency of the formulation. These shock-capturing methods were in the context of entropy variables. In a 1991 ASME paper [10], the original compressible-flow SUPG method, now called "$(\mathrm{SUPG})_{82}$", was supplemented with a very similar shock-capturing term, which included a shock-capturing parameter that is now called "δ_{91}". The shock-capturing parameter was derived from the

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one given in [7] for the entropy variables. It was shown in that, with the added shock-capturing term, $(\text{SUPG})_{82}$ was very comparable in accuracy to $(\text{SUPG})_{82}$ recast in entropy variables. In the 2D inviscid-flow test computations reported in [11] soon after that, $(\text{SUPG})_{82}$ and $(\text{SUPG})_{82}$ recast in entropy variables yielded indistinguishable results. Following these works, references [9,35] generalized discontinuity-capturing methods to arbitrary solution-variable sets. Further developments include the discontinuity-capturing directional dissipation (DCDD) stabilization for incompressible flows [28,36] and the YZβ shock capturing [28–32,37–41], which is based on a scaled residual and has a parameter (β) that controls the degree of shock smoothness. Numerical experiments in [30–32] demonstrated that these new discontinuity capturing techniques are relatively simple to implement and give results of comparable or even improved accuracy relative to earlier approaches. A concise summary of stabilized methods and discontinuity-capturing techniques for compressible flows may be found in a recent review article [42] and references therein.

In this paper, we make use of SUPG stabilization and discontinuity capturing to develop a novel numerical formulation for the Navier–Stokes equations of compressible flows in the Arbitrary Lagrangian–Eulerian (ALE) frame [43] suitable for moving-domain simulations. Early developments in stabilized ALE-based finite-element methods for compressible flows may be found in [44–46]. In the present effort, we introduce several improvements to the existing formulations, as well develop new techniques, such as weakly enforced essential boundary conditions and sliding-interface formulations, that enlarge the scope and applicability of moving-domain, finite-element-based compressible flow formulations.

Weakly enforced no-slip boundary conditions [47] are imposed on solid surfaces in order to avoid excessive resolution of thin, and often turbulent, boundary layers. Weak imposition of essential boundary conditions in the sense of Nitsche's method [48] for incompressible flows was first introduced in [47], and further refined in [49,50]. The most distinguishing feature of this method is the added flexibility to allow the flow to slip on the solid surface in the case when the wall-normal mesh size is relatively large [50–52]. This feature allows one to achieve good accuracy on relatively coarse boundary-layer meshes. Weakly enforced boundary conditions have been successfully applied to simulations of wall-bounded turbulent flows [49,50] and wind turbines [52–55]. More recently, weak enforcement of no-slip conditions was developed and applied in the context of immersogeometric analysis [56–59], which led to solutions of higher-order accuracy on non-boundary-fitted meshes. In the present work, we propose an extension of weakly enforced essential boundary conditions in the context of compressible flows, which brings the aforementioned advantages to this important area of computational fluid mechanics.

The sliding-interface formulation for incompressible flows was introduced in [60] for simulating flows with objects in relative motion. The formulation was comprehensively studied and refined in [54,55], mostly with application to wind turbines. The sliding-interface formulation may be interpreted as a Discontinuous Galerkin method [61], where the basis functions are continuous inside the interior of subdomains but not at the sliding interface. In the incompressible-flow regime, the sliding-interface formulation was recently extended to the space–time (ST) variational multiscale (VMS) method [62–69], and the extension is called the "ST Slip Interface (ST-SI)" method [70–76]. In this work, we develop a compressible-flow counterpart of the sliding-interface formulation.

This paper is organized as follows. In Section 2, we develop a complete numerical formulation of the Navier–Stokes equations of compressible flows. In Section 3, we compute several 2D and 3D examples to verify and validate the different constituents of our compressible-flow numerical methodology. We focus on a broad range of Reynolds and Mach numbers to illustrate the robustness of the numerical formulation. In Section 4, we apply the methods developed to simulate flow inside a gas turbine stage, illustrating the potential of our methods to support design for real engineering systems through high-fidelity aerodynamic analysis. In Section 5, we draw conclusions.

2. Numerical methodology

2.1. Governing equations of compressible flows

2.1.1. Preliminaries

The Navier–Stokes equations of compressible flows are often expressed using a vector of conservation variables $\tilde{\mathbf{U}}$ defined as

$$\tilde{\mathbf{U}} = \begin{bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho e_{\text{tot}} \end{bmatrix}, \tag{1}$$

where ρ is the density, u_i is the ith velocity component, $i = 1, \ldots, d$, where $d = 2$ or 3 is the space dimension, and $e_{\text{tot}} = e + \|\mathbf{u}\|^2/2$ is the fluid total energy density, where e is the fluid internal energy density and $\|\mathbf{u}\|$ is the velocity magnitude.

We also introduce a vector of primitive variables based on pressure or the pressure-primitive variables $\mathbf{Y}$ defined as

$$\mathbf{Y} = \begin{bmatrix} p \\ u_1 \\ u_2 \\ u_3 \\ T \end{bmatrix}, \tag{2}$$

where p is the pressure and T is the temperature. Pressure, density, and temperature are related through an equation of state. Here we make use of the ideal gas equation of state, which may be written as

$$p = \rho R T, \tag{3}$$

where R is the ideal gas constant. Furthermore, we assume a calorically perfect gas and define the fluid internal energy density as

$$e = c_v T, \tag{4}$$

where $c_v = R/(\gamma - 1)$ is the specific heat at constant volume and γ is the heat capacity ratio.

Throughout the paper we use $(\cdot)_{,t}$ to denote a partial time derivative holding the spatial coordinates $\mathbf{x}$ fixed, and we use $(\cdot)_{,i}$ to denote the spatial gradient.

2.1.2. Strong form

The Navier–Stokes equations of compressible flows, which express pointwise balance of mass, linear momentum, and energy, may be written in terms of $\tilde{\mathbf{U}}$ as

$$\tilde{\mathbf{U}}_{,t} + \tilde{\mathbf{F}}^{\text{adv}}_{i,i} = \tilde{\mathbf{F}}^{\text{diff}}_{i,i} + \tilde{\mathbf{S}}, \tag{5}$$

where $\tilde{\mathbf{F}}^{\text{adv}}_i$ and $\tilde{\mathbf{F}}^{\text{diff}}_i$ are the vectors of advective and diffusive fluxes, respectively, and $\tilde{\mathbf{S}}$ is the source term. The residual of the compressible-flow equations may be defined as

$$\widetilde{\mathbf{Res}} = \tilde{\mathbf{U}}_{,t} + \tilde{\mathbf{F}}^{\text{adv}}_{i,i} - \tilde{\mathbf{F}}^{\text{diff}}_{i,i} - \tilde{\mathbf{S}}. \tag{6}$$

We further split the advective flux into $\tilde{\mathbf{F}}^{\text{adv}}_i = \tilde{\mathbf{F}}^{\text{adv}\backslash p}_i + \tilde{\mathbf{F}}^{p}_i$. The aforementioned fluxes are defined as

$$\tilde{\mathbf{F}}^{\text{adv}\backslash p}_i = \begin{bmatrix} \rho u_i \\ \rho u_i u_1 \\ \rho u_i u_2 \\ \rho u_i u_3 \\ \rho u_i \left(e + \|\mathbf{u}\|^2/2\right) \end{bmatrix}, \tag{7}$$