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IFAC PapersOnLine 50-1 (2017) 7076–7081

Optimal Sensor Location and Mobile Sensor Crowd Modeling for Environmental Monitoring

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Abstract: In this paper, the optimal sensor location problem is first discussed for environmental monitoring of physical phenomena governed by some advection-diffusion partial-differential equations. In particular, the exact derivation of the observability Gramian of an advection-diffusion PDE is investigated. Based on the optimality criteria derived from this analysis, a conservation law governing the behavior of a crowd of mobile sensors is proposed to ensure convergence of the sensor density towards an optimal location. The monitoring of pollution on a 2D domain is the case study used throughout the paper to illustrate the effectiveness of the proposed approach.

Keywords: Pollution monitoring, optimal sensor location, sensor crowds, collective behavior, conservation laws, approximate observability.

1. INTRODUCTION

Mobile sensors in networks (such as coordinated fleet of drones) represent an attractive way to ensure monitoring or tracking of time-varying spatially-distributed environmental phenomena (weather events, wildfires, air, soil, river or sea pollution ...). Presently monitoring systems are still mostly based on static networks of sensors, see Ghanem (2004) for instance. However the use of mobile sensors can potentially provide more flexibility in collecting distributed data when the conditions are changing.

The navigation of mobile sensors for environmental monitoring is classicaly based on concentration gradient and flow direction to track pollutant sources (Cortes (2004)). In Demetriou (2011); Ucinski (2005), the authors derive a stable distributed-parameter state observer by using measurements from some mobile sensors which have to be controlled to satisfy this goal. Following the same idea, in Georges (2013), a nonlinear conservation law is proposed to model the collective behavior of a mobile sensor continuum used for pollution monitoring purpose. The sensor density reaches an equilibrium corresponding to the necessary conditions for optimality (observability maximization or trade-off between observability maximization and obstacle avoidance in the presence of obstacles).

In the present paper the later approach is improved in the sense that the proposed conservation law derivation is now based on the exact infinite-dimensional observability gramian of an advection-diffusion PDE in 2D, which can be easily computed, rather than on an approximate finite-dimensional observability gramian derived from a finite-dimensional model obtained thanks to a Galerkin method. The application field still concerns all the physical phenomena governed by advection-diffusion PDEs. The here-proposed approach can be closely related to the work by Privat et al (2015) for the wave equation.

The main objective remains to get an optimal configuration of the sensors suitable to enhance the performance of state observers designed for estimation or prediction of the distributed pollution dynamics, by mainly improving sensor measurement output sensitivity with respect to the initial state distribution.

The organization of the paper is now as follows: In section 2, the advection-diffusion PDE governing the pollution dispersion phenomena is recalled. Section 3 is devoted to both the exact derivation of both the infinite-dimensional Gramian of a linear advection-diffusion PDE and an observability criteria used for the optimal location of sensors. In section 4, a new nonlinear conservation law governing a crowd of mobile sensors is derived using the results of section 3. In section 5, a case study is investigated, which demonstrate the effectiveness of the methodology. Finally the paper ends with some conclusions and perspectives.

2. THE ADVECTION-DIFFUSION PDE (ADPDE)

The pollution dispersion phenomena may be well modeled (see Zannetti (1990) for instance), in the case of air pollution) on a domain Ω by the following advection-diffusion partial differential equation:

$$\frac{\partial u}{\partial t}(x,t) + V(x,t)\cdot\nabla u(x,t) = k\triangle u(x,t) - \beta u(x,t) + S(x,t) \quad (1)$$

where $x \in \Omega \subset \mathbb{R}^N$, with $N = 1, 2$ or 3, $u(x,t)$ is the concentration of a chemical species (the pollutant), $V(x,t)$ is a vector of flow velocities which is supposed to be known (through measurements or computation of other PDEs,

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2017.08.1355

such as the Saint-Venant equations governing the dynamics of open-channel hydraulic systems or meteorological models in the case of air pollution), $k > 0$ is a constant diffusion coefficient which is supposed to be known, $\beta > 0$ is a reaction coefficient, and source term $S(x, t)$ acts in the domain Ω. ∇ and $\triangle$ stand for the gradient and the Laplacian respectively. " $\cdot$ " denotes the standard scalar product.

In this paper, only the $2D$ case will be studied. This means that there is no concentration gradient according to the vertical coordinate. It should be however pointed out that the methodology proposed here can be easily extended to the $3D$ case.

A rectangular domain $\Omega = \{(x, y) : \ 0 \leq x \leq L, \ 0 \leq y \leq H\}$ is introduced, together with the following initial and boundary conditions [1]

$$u(x, y, t = 0) = u_0(x, y), \tag{2}$$

and

$$u(0, y, t) = u(L, y, t) = 0, \tag{3}$$
$$u(x, 0, t) = u(x, H, t) = 0. \tag{4}$$

Now we use the following assumptions in what follows:

Assumption 1: Velocity field $V(x, y, t)$ is supposed to be uniform over domain Ω: $V(x, y, t) = V(t)$.

Assumption 2: The source term $S(x, y, t)$ is known.

Assumption 3: The time-varying velocity $V(t)$ is replaced by a mean velocity $\bar{V}$ defined over a finite time interval $[0, T]$:

$$\bar{V} = \frac{1}{T}\int_0^T V(t)dt. \tag{5}$$

This assumption means that the velocity field $V(t)$ is available through measurements or predictive computation over $[0, T]$.

Even if the attention will be paid to 2D air pollution in the paper, many other applications are covered by the methodology proposed here: Monitoring of pollution advection-diffusion in 1D or 2D shallow water systems (rivers, lakes, estuaries or seas), underseas pollution or groundwater monitoring.

3. OBSERVABILITY ANALYSIS OF THE ADPDE

3.1 Background on Observability Gramian of Linear PDEs

For linear time-invariant finite-dimensional systems:

$$\dot{x} = Ax \tag{6}$$
$$y = Cx \tag{7}$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^p$, the so-called "transient observability function" is defined as (see Brockett (1970) for instance)

[1] Neumann Boundary Conditions can also be used without restriction.

$$L_o(X, T) = \frac{1}{2}\int_0^T \|y(t)\|^2 dt, \ x(0) = X, \tag{8}$$

that is the output energy generated by any initial state $X \in R^n$ in the time interval $[0, T]$. L_o may be rewritten as

$$L_o(X, T) = \frac{1}{2}X^T W(T)X, \tag{9}$$

with

$$W(T) = \int_0^T e^{A^T t} C^T C e^{At} dt. \tag{10}$$

$W(T)$ is the so-called "transient observability Gramian" matrix.

A necessary and sufficient condition for observability (resp. detectability) of the pair (C, A) is that there exists $\forall t \in [0, T]$, a positive definite (resp. non negative definite) symmetric matrix $W(t)$, solution to the following differential Lyapunov equation:

$$-\dot{W}(t) + A^T W(t) + W(t)A = -C^T C, \quad W(0) = 0. \tag{11}$$

The computation of this Lyapunov equation provides the observability Gramian at time T.

In the case of asymptotically stable observable (resp. detectable) linear systems defined by the pair (C, A), L_o is finite when $T \to +\infty$ and $\lim_{T\to+\infty} W(T) = W^\infty$. W^∞ is obtained as the unique positive definite (resp. non negative definite) solution to the Lyapunov problem:

$$A^T W + WA = -C^T C. \tag{12}$$

It follows that $W(T)$ or W^∞ can be used as a measure of the observability degree of the system, since the eigenvalues of W or W^∞ represent the sensitivity of output y with respect to each component of any initial state $x(0) = X$. Indeed, if the sensitivity of y to the intial state (denoted $\partial_{x_0} y^T(t)$) is given by

$$\frac{d}{dt}(\partial_{x_0} x^T(t)) = A\partial_{x_0} x^T(t), \ \partial_{x_0} x^T(0) = I_d, \tag{13}$$
$$\partial_{x_0} y^T(t) = C\partial_{x_0} x^T(t), \tag{14}$$

where $\partial_{x_0} x^T$ denotes the sensitivity matrix of x with respect to initial state x_0, then the observability Gramian can be recovered as a Fisher Information Matrix (FIM):

$$\partial_{x_0} y^T(t) = Ce^{At} \tag{15}$$
$$\int_0^T \partial_{x_0} y(t)\partial_{x_0} y^T(t)dt = \int_0^T e^{A^T t} C^T C e^{At} dt = W(T). \tag{16}$$

It is interesting to notice that such a sensitivity analysis can be useful to compute the FIM of the sensitivity of y to an unknown constant or slowly time-varying input (source) u for the state-space system:

$$\dot{x}(t) = Ax(t) + Bu, \ x \in \mathbb{R}^N, \ u \in \mathbb{R}^m, \ x(0) = x_0$$
$$y(t) = Cx(t), \ y \in \mathbb{R}^p.$$

Applying the previous sensitivity computation leads to: