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# Relation between higher order comoments and dependence structure of equity portfolio

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ABSTRACT

We study a relation between higher order comoments and dependence structure of equity portfolio in the US and UK by relying on a simple portfolio approach where equity portfolios are sorted on the higher order comoments. We find that beta and coskewness are positively related with a copula correlation, whereas cokurtosis is negatively related with it. We also find that beta positively associates with an asymmetric tail dependence whilst coskewness negatively associates with it. Furthermore, two extreme equity portfolios sorted on the higher order comoments are closely correlated and their dependence structure is strongly time-varying and nonlinear. Backtesting results of value-at-risk and expected shortfall demonstrate the importance of dynamic modeling of asymmetric tail dependence in the risk management of extreme events.

## 1. Introduction

The Fama–French factor model (Fama and French, 1993) is a monumental turning point in the modern asset pricing literature. Recently, Christoffersen and Langlois (2013) study how an extreme dependence structure associates with the Fama–French factors and address its role in broad area of finance. They also emphasize the importance of copula modeling for the extreme dependence structure. On the other hand, there is a group of researchers supporting the importance of higher order comoments in asset pricing (Harvey and Siddique, 2000; Dittmar, 2002; Bakshi et al., 2003; Ang et al., 2006; Guidolin and Timmermann, 2008; Chabi-Yo, 2012; Maheu et al., 2013; Chabi-Yo et al., 2014a). Although those are less popular than the Fama–French factors among practitioners, those have been rigorously developed from theoretical perspectives. Hence, it is academically interesting to study how the extreme dependence structure is related with the higher order comoments and address their implications in finance.

A few papers address a relation between the higher order comoments and the tail dependence of equity portfolio. They show that it has a close relationship with, not only beta, but also coskewness. For example, Garcia and Tsafack (2011) show that a strong dependence in lower returns creates a large negative coskewness in their international bond and equity market portfolio analysis. Chabi-Yo et al. (2014b) also show that a strong lower tail dependence creates a large negative coskewness. In addition they show that beta is monotonically increasing with respect to the lower tail dependence. From these studies, we are able to draw an inference that the tail dependence is a key driver to create the higher order comoments of the equity portfolio. Thus our first research question is how the higher order comoments associate with the dependence structure of the equity portfolio. We approach our research question by relying on a simple portfolio approach. Specifically, we sort equities into portfolios based on the size of the higher order

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comoment, i.e., from low beta (coskewness, cokurtosis) to high beta (coskewness, cokurtosis), and test patterns of a copula correlation or an asymmetric dependence across the characteristic-sorted portfolios.

We find that there are statistically significant patterns between the higher order comoments and the dependence structure of the equity portfolio. First, beta and coskewness are positively related with the copula correlation whilst cokurtosis is negatively related with it. Second, we find the asymmetry that the lower tail dependence is stronger than the upper tail dependence for all portfolios. Third, beta is positively related with the asymmetric tail dependence, whereas coskewness is negatively related with it.

Our second research question is what economic implication is contained by the relation between higher order comoments and the dependence structure of the equity portfolio. We find its implication from risk management perspectives. We often long and short two extreme portfolios to hedge their risk. Thus the higher order comoment risk can be also hedged by buying and selling two extreme beta (coskewness, cokurtosis) portfolios, i.e., *b*uying *m*inus *s*elling (BMS) portfolio. However, if our inference is correct in the first research question, the higher order comoments are unable to be key inputs for the risk management of extreme events. Rather, a key driver is the tail dependence which creates the higher order comoments. To investigate our second research question, we apply backtesting tools to alternative models: dynamic copula models, multivariate GARCH model and univariate model. The dynamic copula models fully incorporate the dependence structure of two extreme portfolios whilst the multivariate GARCH model takes into account only the second order comoment. The univariate model considers neither the tail dependence nor the second order comoment.

The backtesting results strongly support the importance of modeling the time-varying and asymmetric dependence of the BMS portfolio. First, we find that the dependence structure of the BMS portfolio is strong, time-varying and asymmetric for all characteristic-sorted portfolios. Second, both the multivariate GARCH model and the univariate model significantly underforecast value-at-risk (VaR) and expected shortfall (ES). Third, the dynamic copula models show not only robust coverage ability but also statistical accuracy for VaR and ES.

Besides two important research questions, we develop a generalized dynamic asymmetric copula. Our proposed model takes into account two important characteristics of equity *t* portfolios; a time-varying dependence and an asymmetric tail dependence. First, we employ a generalized hyperbolic skewed *t* distribution (see Demarta and McNeil, 2005) to capture the asymmetric dependence structure. Second, the time-varying copula correlation is implied by the generalized autoregressive score (Creal et al., 2013). Hence, our proposed model can cover for the most types of the dependence structure revealed by the equity portfolios. We apply our copula to estimating the dependence structure in our analysis.

Our study makes three contributions. First, we provide comprehensive analysis on the relation between higher order comoments and the dependence structure of the equity portfolio. We find the striking evidence that the higher order comoments are closely related with the dependence structure of the equity portfolio in the US and UK. Second, we demonstrate the importance of modeling the time-varying and asymmetric dependence of the BMS portfolio in the risk management of extreme events. The backtesting results show that the ignorance of dependence asymmetry and dynamics is costly in the risk management. Third, we propose the generalized dynamic asymmetric copula by combining the generalized hyperbolic skewed *t* distribution and the generalized autoregressive score. Our proposed copula performs well in estimating the dependence structure of the BMS portfolio and forecasting both VaR and ES.

The remainder of this paper is organized as follows. In Section 2, we detail the way we employ for the portfolio construction and the dynamic asymmetric copula we propose. The data used in the paper and the descriptive statistics are in Section 3. In Section 4, we focus on the analysis of the relation between the higher order comoments and the dependence structure. In Section 5, we analyze the role of the dependence structure of the BMS portfolio in the forecasting based risk management application. In Section 6, we perform the robustness check to different estimation periods. Finally, conclusions are given in Section 7.

## 2. Methodology

In this section, we detail a way we employ for the portfolio construction and models we use in this paper.

### 2.1. Portfolio construction

A return on an asset is defined as the first difference of the log price, $r_t = \log P_t - \log P_{t-1}$. We construct portfolios sorted on beta, coskewness and cokurtosis, respectively. Following the definition of Bakshi et al. (2003) and Conrad et al. (2013), we define the market beta, coskewness and cokurtosis by

$$BETA_{i,t} = \frac{\mathbb{E}[(r_{i,t} - \mathbb{E}[r_{i,t}])(r_{m,t} - \mathbb{E}[r_{m,t}])]}{Var(r_{m,t})}, \tag{1}$$

$$COSK_{i,t} = \frac{\mathbb{E}[(r_{i,t} - \mathbb{E}[r_{i,t}])(r_{m,t} - \mathbb{E}[r_{m,t}])^2]}{\sqrt{Var(r_{i,t})}Var(r_{m,t})}, \tag{2}$$

$$COKT_{i,t} = \frac{\mathbb{E}[(r_{i,t} - \mathbb{E}[r_{i,t}])(r_{m,t} - \mathbb{E}[r_{m,t}])^3]}{Var(r_{i,t})Var(r_{m,t})}. \tag{3}$$

All stocks are sorted on each characteristic above and divided into five groups based on the 20th, 40th, 60th and 80th percentiles. We