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On reliability improvement program for second-hand products sold with a two-dimensional warranty

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ABSTRACT

The growing market for second-hand products facilitates the acceptance of warranty provided by dealers to promote sales and provide assurance for customers. Offering warranty results in additional cost due to warranty claims servicing. Such cost can be reduced through upgrade and preventive maintenance (PM) that improve the reliability of the product. Reliability improvement is worthwhile only if the cost incurred is less than the reduction in the warranty cost. For repairable second-hand products sold with a two-dimensional warranty, this paper investigates the worthiness of reliability improvement from the dealer's viewpoint. A new modeling approach is proposed and studied, allowing the same product to have distinct degradation processes due to customer usage intensity variation. The effects of customer usage heterogeneity, upgrade and PM actions on the product reliability and the corresponding expected cost to the dealer are characterized. Optimal reliability program including the upgrade degree and PM policy is derived so that the expected total cost of the second-hand product to the dealer can be minimized. Numerical examples are utilized to illustrate the applicability of the proposed model. In addition, some practical implications from a sensitive analysis regarding the important parameters are elaborated.

1. Introduction

Rapid technological advances coupled with customer demand have prompted new products launched into the market at an ever increasing pace. The sales often occur with a trade-in, which results in a continuously growing and competitive market for second-hand products. For example, the trade volume for second-hand cars in China rose to 9.42 million in 2015, which accounted for 43.79% of that for new cars [2,35]. Reliability is one of the most important characteristic of product for consumers in making their purchase decisions among different options [1]. Even well-developed products are unreliable in the sense that they deteriorate with age and/or usage, and ultimately fail to perform their required functions under specified operating conditions [27]. Customer concern over the reliability at the end of first life may impede the development of second-hand product market, especially when lacking information on the usage intensity and maintenance history of the first user [23]. To remedy this uncertainty and promote sales, dealers offer warranty as a significant instrument to assure consumers about product performance. Such warranty is a contractual agreement offered by the dealer to carry out corrective actions to rectify any problems (such as the item not performing as expected, failures of components, etc.) that the customer experiences within the stated warranty coverage. A detailed review of various issues related to warranty policies can be referred to Murthy and Djamaludin [25].

Offering warranty incurs additional costs that are mainly associated with maintenance performed under warranty in case of failures. For a second-hand product, in addition to the warranty terms and servicing strategy, the expected warranty cost incurred to a dealer is closely related to product reliability which in turn depends on its unique past age and/or usage and maintenance history [9]. Inadequate reliability causes higher warranty cost resulting from product failures and need to be addressed. One feasible way to improve the reliability is through initiatives such as upgrade and preventive maintenance (PM). An upgrade action is implemented prior to the sale of the second-hand product to improve it to a better functional state and reduce the likelihood of following failures. PM actions are usually scheduled and carried out to control the deterioration process leading to a failure of the product. Main works dealing with PM modeling for products sold with warranty are reflected in the available publications [6,10,11,16,17,21,22,28,30,34,39–42].

It is worthwhile for the dealer to carry out reliability improvement only if the expected savings in the warranty cost exceed the extra cost in-

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curred. An amount of academic research has been focused on reliability improvement modeling and failure analysis of second-hand products. To name a few, Chattopadhyay and Murthy [7] developed two warranty cost models and decided on the optimal reliability improvement strategies for used products sold with free repair warranty (FRW) policy. Shafiee et al. [32,33] dealt with several cost-benefit models to derive the optimal upgrade scheme under given cost structures. Su and Wang [36] proposed a stochastic model to jointly optimize the upgrade level and PM policy so as to maximize the dealer's expected profit. Considering the possibility of performing PM during warranty period, Darghouth et al. [12] determined the optimal reliability improvement level for second-hand production equipment sold with FRW policy.

The above researches are limited to the case of one-dimensional warranty under which only the age or usage is restricted. Under two-dimensional warranty, product failures are indexed by two scales - age and usage. To the best of our knowledge, there are currently five reported publications related to two-dimensional warranty policies for second-hand products. Using one-dimensional approach to model failures [18,19], Chattppadhyay and Yun [8], and Shafiee et al. [31] estimated the expected warranty cost for second-hand products without considering reliability improvement. Su and Wang [38] developed a stochastic model to determine the optimal upgrade policy that minimized the dealer's expected total cost for used products sold with two-dimensional warranty. Sarada and Mubashirunnissa [29] dealt with warranty cost analysis for second-hand products through bivariate failure modeling approach [3,4,24,26], under which the product age and usage were positively quadrant dependent. Afterwards, Su and Wang [37] developed an upgrade model for repairable used products sold with two-dimensional warranty to design flexible warranty contracts. Most of these works tacitly assume that the first and latter users of the second-hand product are homogeneous in usage intensity. This implies the lifetime of the product before and after its resale can be modeled by the same failure distribution function. While it may be not quite the case. Product degradation is a stochastic process in which individuals age differently under different environment [15]. During its past life, the product has been used under observed usage intensity by its first user. Once the product is switched to another user, the usage pattern may change accordingly and affect the failure process during its remaining lifetime. Ignoring the effect of consumer usage heterogeneity on the product deterioration may result in inaccurate cost estimation, and an inferior reliability improvement decision.

In view of the deficiencies, we proposes in this paper a new reliability improvement model for second-hand products sold with a two-dimensional warranty policy. Introducing a variable "*effective age*", the effect of usage intensity variation on the product degradation is modeled through Accelerated Failure Time (AFT) approach. Based on the setting, the modified failure intensity function is derived by taking the consumer usage heterogeneity into consideration. After that, several cost models are developed considering different reliability improvement options. The main objective is to determine the optimal reliability improvement program so as to minimize the dealer's expected total cost. The proposed model hopes to provide practical guidelines for reliability analysts and dealers to effectively design optimized reliability improvement programs and achieve cost reduction of second-hand products.

The remainder of this paper is organized as follows. In the next section, model assumptions and notations are described. In Section 3, reliability improvement models under four options are developed. To do this, the failure process of product is derived in terms of age and usage rate. Then, the effects of upgrade and PM on the product reliability are characterized respectively. In Section 4, the expected total cost under each option is estimated, and the mathematical cost-based optimization model is proposed. For a special case of Weibull lifetime distribution, illustrative numerical examples are presented in Section 5. Following that, in Section 6, we articulate practical implications and develop several guidelines considering the use of reliability improvement. Finally, in Section 7 the concluding remarks and a brief discussion of topics for further research are given.

2. Model assumptions and notations

The assumptions and notations used in this paper are given as follow.

Model assumptions:

- The specified second-hand product is repairable and sold with a two-dimensional non-renewing free repair warranty (NFRW) policy.
- Information on the past age, usage and operational history of the product is available to the dealer.
- During its first life, the product has been minimally repaired all the way.
- Upgrade is performed by the dealer prior to the sale of the product.
- All failures occurring under warranty are statistically independent and minimally repaired by the dealer for free.
- Every item failure results in an immediate warranty claim and all claims are valid.
- The periodic PM policy is planned and implemented within the warranty coverage. No PM action is performed at the expiration of the warranty.
- The durations of the upgrade, each PM and repair are small compared to the mean time between two successive failures, and so can be negligible.

Model notations:

K, L	Past age and usage of the second-hand product
W, U	Age and usage limits of a two-dimensional warranty policy
Ω, Φ	Original and modified warranty coverage
x, u	Product age and total usage
r_0, R	Nominal usage rate and the random customer usage rate
$g(r)$	Density function for R
r_1	Usage rate during the product's past life
X_0 (X_r)	Time to first failure under usage rates r_0 (r)
$F(x\|r_0)$	Failure distribution function with nominal usage rate r_0
$F(x\|r)$, $f(x\|r)$	Conditional failure distribution function and density function with usage rate $R = r$
$h(x\|r)$, $h(x\|r_0)$	Hazard function associated with $F(x\|r)$ and $F(x\|r_0)$
z	Accelerated coefficient for failures ($z \geq 1$)
$\lambda(x\|r)$, $\lambda(x\|r_0)$	Failure intensity functions under usage rates r and r_0
θ	Upgrade degree (decision variable, $0 \leq \theta \leq 1$)
$\lambda(x, \theta\|r)$	Failure intensity function under usage rate r after upgrade
n, m	Number of PM actions and PM level (decision variables)
w_j	Time instant for carrying out the jth PM, $1 \leq j \leq n$
v_j	Virtual age of the product after the jth PM, $1 \leq j \leq n$
$v_j(x\|r)$	Virtual age of the product at time x after jth PM under usage rate r, $1 \leq j \leq n$
$\lambda_m(x, \theta\|r)$	Failure intensity function under usage rate r considering both upgrade and PM
$\lambda_m(x\|r)$	Failure intensity function under usage rate r with PM only
τ, μ	Expected age and usage intervals of PM
C_r	Cost of minimal repair in case of a warranty failure
$C_u(\theta)$	Upgrade cost with upgrade degree θ
$C_p(m)$	Cost of each PM with maintenance level m
$\phi = \{\theta, n, m\}$	Set of parameters of the reliability improvement program
$TC_\Omega(\phi; K, L)$	Expected total cost of the second-hand product to the dealer with past age K and usage L

3. Model formulation

3.1. Warranty policy and Item failures

The second-hand product with past age K and usage L, is sold by the dealer with a two-dimensional warranty. The rectangular warranty region is considered here and defined by the set $\Omega = [K, K + W] \times [L, L + U]$, as shown in Fig. 1. All failures under warranty are rectified by the dealer at no cost to the consumer. The warranty expires when either the product age reaches the limit $K + W$ or the total usage exceeds the level $L + U$, whichever occurs first.

Under two-dimensional warranty, item failures can be viewed as random events occurring within the warranty region, and modeled by a