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Set-valued discrete-time sliding-mode control of uncertain linear systems

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Abstract: This paper focuses on the discrete-time sliding-mode control problem, that is, given an uncertain linear system under the effect of external matched perturbations, to design a set-valued control law that achieves the robust regulation of the plant and at the same time reduces substantially the chattering effect in both the input and the sliding variables. The cornerstone is the implicit Euler discretization technique together with a differential inclusion framework which allow us to make a suitable selection of the control values that will compensate for the disturbances. Numerical examples confirm the effectiveness of the proposed methodology.

Keywords: Differential inclusions, robust control, sliding-mode control, discrete-time systems, linear uncertain systems, Lyapunov stability.

1. INTRODUCTION

There exists an extensive literature on discrete-time sliding-mode control which, at this point, can be divided into two groups. In one group we have the works that rely on discontinuous control actions, as for example Bartoswewicz [1998], Galias and Yu [2007], Gao et al. [1995], Kaynak and Denker [1993], Spurgeon [1991]. The sliding-mode control law is discretized using an explicit Euler technique and is limited by the condition that the ideal sliding-mode is never reached, leading to concepts such as *quasi sliding*, a term that refers to the fact that the system trajectories will ultimately belong to a boundary layer of the sliding manifold even in the absence of disturbances. The main problem with the discontinuous control approach is the susceptibility to the appearance of chattering. Indeed, at a point of discontinuity the control law cannot take values lying between its different limits, so a high frequency switching becomes necessary for maintaining the system in the sliding phase [Utkin 1992]. It is thus not surprising to see considerably high levels of chattering in these schemes.

The central idea among the second group of controllers is that, similar to the differential inclusions described in the work of Filippov and Arscott [1988], the discrete-time system should be governed by a difference inclusion, not a difference equation [Acary and Brogliato 2010, Acary et al. 2012, Huber et al. 2016b,c]. These works are based on the use of set-valued control laws for which a selection compensating the matched disturbances is possible.

In practical terms, the difference between both approaches lays on the type of discretization used. Whereas the former group employs an explicit Euler discretization, the second one employs an implicit one. In the latter case, the resulting controller turns out to be Lipschitz continuous, which results in a substantial reduction of chattering, Huber et al. [2016b,c], Wang et al. [2015].

The present work falls into the second group and is dedicated to the study of uncertain systems, i.e., we consider the case where the system matrices are uncertain. The class of uncertainty considered is large enough to embrace parametric uncertainty as well as nonlinear unmodeled dynamics and external perturbations. It is also worth remarking that the works by Acary and Brogliato [2010], Acary et al. [2012], Huber et al. [2016b,c] do not consider uncertainty in the system parameters.

The paper is organized as follows: Section 2 sets the notation and recalls some concepts from convex analysis. Section 3 presents, very shortly, the design of continuous-time sliding-mode controllers for systems with model uncertainty and external matched disturbances. Section 4 constitutes the main body of the paper. Here, the methodology design of discrete-time sliding mode controllers is presented together with well-posedness and stability results. Finally, Section 5 shows the effectiveness of the proposed controller and its superior performance when compared against explicit Euler discretization techniques.

2. PRELIMINARIES AND NOTATION

Let $\mathbb{R}^n$ be a n-dimensional linear space, given with the classical Euclidean inner product denoted as $\langle \cdot, \cdot \rangle$ and the corresponding norm $\| \cdot \|$.

Definition 1. Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be a proper, convex, lower semicontinuous function. The subdifferential of f at $x \in \operatorname{Dom} f$ is given by the set

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$$\partial f(x) := \{\zeta \in \mathbb{R}^n \mid \langle \zeta, \eta - x\rangle \leq f(\eta) - f(x), \text{ for all } \eta \in \mathbb{R}^n\}.$$

Definition 2. Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be a proper, convex, lower semicontinuous function. The proximal map $\mathrm{Prox}_f : \mathbb{R}^n \to \mathbb{R}^n$ is the unique minimizer of $f(w) + \frac{1}{2}\|x - w\|^2$, that is,

$$f(\mathrm{Prox}_f(x)) + \frac{1}{2}\|x - \mathrm{Prox}_f(x)\|^2 = \min_{w\in\mathbb{R}^n} \left\{ f(w) + \frac{1}{2}\|x - w\|^2 \right\}.$$

Note that for $\Psi_{\mathcal{C}}$, the indicator function of the set $\mathcal{C}$, the proximal map corresponds to the well-know projection operator, see Hiriart-Urruty and Lemaréchal [1993]. The following result, extracted from [Bauschke and Combettes 2011, Proposition 12.26], establishes a link between the two former concepts.

Proposition 3. Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be a proper, convex, lower semicontinuous function. Then, $p = \mathrm{Prox}_f(x)$ if, and only if, $x - p \in \partial f(p)$.

Remark 4. It follows from Proposition 3 that the map $(I + \alpha\partial f)^{-1}$ is singled valued. More specifically, $\mathrm{Prox}_{\alpha f} = (I + \alpha\partial f)^{-1}$. Indeed, assume that y_i, $i = 1, 2$ are such that $y_i \in (I + \alpha\partial f)^{-1}(x)$. We have, $x - y_i \in \alpha\partial f(y_i)$, $i = 1, 2$. Hence, Proposition 3 gives $y_1 = y_2 = \mathrm{Prox}_{\alpha f}(x)$.

In the upcoming discussion the conjugate function $f^\star$ of a proper function will play an important role. Here we recall its definition.

Definition 5. Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$. The conjugate of f is,

$$f^\star(z) := \sup_{x\in\mathbb{R}^n} \{\langle z, x\rangle - f(x)\}.$$

Theorem 6. (Moreau's decomposition). Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be a proper, convex, lower semicontinuous function and let $\alpha \in \mathbb{R}$ be strictly positive. Then, for any $x \in \mathbb{R}^n$, the following identity holds:

$$x = \mathrm{Prox}_{\alpha f}(x) + \alpha\, \mathrm{Prox}_{f^\star/\alpha}(x/\alpha).$$

Along this paper we denote the identity matrix in $\mathbb{R}^{n\times n}$ as I_n. The set $\mathcal{B}_n := \{x \in \mathbb{R}^n \mid \|x\| < 1\}$ represents the unit open ball with center at the origin in $\mathbb{R}^n$ with the Euclidean norm. The interior, closure, and boundary of a set $\mathcal{S} \subset \mathbb{R}^n$ are denoted as $\mathrm{int}\,\mathcal{S}$, $\mathrm{cl}\,\mathcal{S}$, and $\mathrm{bd}\,\mathcal{S}$ respectively.

3. A QUICK REVIEW OF CONTINUOUS-TIME SLIDING-MODE CONTROL

We begin with a quick look at the continuous-time sliding-mode control problem. To this end, let us consider the uncertain plant

$$\dot{x} = (A+\Delta_A(t, x))x(t)+B(u(t)+w(t, x)),\ x(0) = x_0, \quad (1)$$

where $x(t) \in \mathbb{R}^n$ represents the state of the system, $u(t) \in \mathbb{R}$ is the scalar control input and $w(t, x) \in \mathbb{R}$ accounts for external disturbances and unmodeled dynamics. The matrices A, Δ_A and B are of the appropriate dimensions. It is assumed that the matrix $\Delta_A(t, x)$ is unknown but is uniformly upper-bounded by

$$\Delta_A(t, x)\Lambda\Delta_A^\top(t, x) < I_n \quad (2)$$

with $\Lambda = \Lambda^\top > 0$ a known matrix. We also make the following standard assumptions.

Assumption 7. The pair (A, B) is stabilizable.

Assumption 8. The disturbance term $w(t, x)$ is uniformly bounded in the $\mathcal{L}^\infty$ sense, that is, there exists $W > 0$ such that $\sup_{t\geq 0} \|w(t, x)\| \leq W < +\infty$.

The first step in the design of sliding-mode controllers consists in fixing the sliding surface $\sigma(x) = 0$ in such a way that the behaviour of the system constrained to the sliding surface satisfies the performance requirements. The second step consists in the design of the control law that will steer the state towards the sliding surface and will maintain it there, even in the presence of model uncertainties and external disturbances. An assumption concerning the sliding surface is the following.

Assumption 9. The matrix $C \in \mathbb{R}^{1\times n}$ is such that the product CB is nonsingular.

The previous assumption ensures the uniqueness of the equivalent control (see, e.g. Utkin et al. [2009]). Namely, by considering the sliding surface as the hyperplane $\sigma = Cx$, the equivalent control is computed from the invariance condition $\dot{\sigma} = 0$ as

$$C(Ax^{\mathrm{eq}} + B(u^{\mathrm{eq}} + w)) + \Delta_A(t, x^{\mathrm{eq}})x^{\mathrm{eq}}) = 0 \Rightarrow$$
$$u^{\mathrm{eq}} = -(CB)^{-1}C\,(Ax^{\mathrm{eq}} + \Delta_A(t, x^{\mathrm{eq}})x^{\mathrm{eq}}) - w.$$

Substitution of the equivalent control into (1) leads to the expression of the dynamics in sliding phase,

$$\dot{x}^{\mathrm{eq}} = \left(I_n - B(CB)^{-1}C\right)(A + \Delta_A(t, x^{\mathrm{eq}}))x^{\mathrm{eq}}, \quad (3)$$

from which it becomes clear that the matrix characterizing the sliding hyperplane plays a role in the reduced system dynamics. There exists many methods for the design of the sliding surface, e.g., LQR design [Utkin 1992, Chapter 9], eigenvalue placement [Utkin et al. 2009, Chapter 7], $\mathcal{H}_\infty$ control [Castaños and Fridman 2006], linear matrix inequalities [Polyakov and Poznyak 2011], see also [Shtessel et al. 2014, Section 2.4.2], among others. Here we relegate the design of the sliding surface in continuous time to the background and focus instead on the discrete-time setting. As mentioned above, the second step consists in designing the set-valued control law that will bring the system into the sliding regime. The design procedure is divided into two steps. Namely, first we compute a control law for the nominal version of (1) (i.e., $\Delta_A \equiv 0$ and $w \equiv 0$) and then the set-valued controller that will provide the necessary robustness. Thus, the control law is set as

$$u = u^{\mathrm{nom}} - \gamma_1(x)\,\mathrm{Sgn}(\sigma), \quad (4)$$

where u^{nom} is a control input for the nominal system and $\gamma_1 : \mathbb{R}^n \to \mathbb{R}_+$ is a control gain. It is worth remarking that the trajectories of the closed-loop (1), (4) will reach the sliding surface $\sigma = Cx$ in finite time, from where the reduced system will go asymptotically to the origin whenever the matrix C is well-designed.

In conclusion, the common methodology design for sliding-mode controllers in continuous time relies on the appropriate design of the matrix C that will make the reduced system asymptotically stable, whereas the set-valued controller will compensate for all the matched disturbances.