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Asymptotic behavior analysis of complex-valued impulsive differential systems with time-varying delays[☆]

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ABSTRACT

In this paper, asymptotic behavior analysis is addressed for a class of nonlinear complex-valued impulsive differential systems with time-varying delays. By establishing two inhomogeneous impulsive delay differential inequalities and applying the theory of matrix measure, the global exponentially attractive set, invariant set and weak-invariant set of the complex-valued impulsive delay differential systems are derived. Examples are provided to illustrate the effectiveness of the proposed results.

1. Introduction

Complex-valued differential systems are an important class of dynamical systems whose solutions and unknown functions are complex-valued functions. These systems are more difficult, complex and challenging than the real-valued differential systems, although they can be considered as an extension of the real-valued ones. Complex-valued differential systems have been drawing more and more attention benefiting from their brand-new applications in quantum mechanics [1–7] and neural network [8–15] among others. By separating the complex-valued differential systems into a real part and an imaginary part and transforming the n-dimensional complex-valued differential systems to the 2n-dimensional real-valued differential systems, then combining the research method of real-valued differential systems, the stability and the asymptotic behavior of the solution for complex-valued differential systems have been studied in [9,16–18]. In spite of the apparent advantage resulting from abundant and powerful tools existing for the real analysis, there are two apparent problems: (i) an explicit separation of a complex-valued function $f(t, z)$ into a real part and an imaginary part is not always possible; and (ii) the dimension of the real-valued system is double that of the complex-valued system, which leads to difficulties in analysis [19]. Therefore, it is necessary to establish an effective method to overcome these problems. Recently, Fang and Sun developed some methods to analyze the complex-valued systems, which retained the complex nature of the systems and investigated their properties on $\mathbb{C}^n$ [19].

On the other hand, impulsive dynamical systems have received much attention from researchers since they provide a natural framework for mathematical modeling of many real world systems whose evolution in time undergo sudden changes [20–22]. These systems have found important applications in various fields, such as dosage supply in pharmacokinetics [21], quantum measurements [23], orbital transfer of satellite [24], sampled-data control systems [25,26] and networked control systems [27]. Massive outstanding achievements both on the theoretical analysis and the application of

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impulsive dynamical systems have been obtained [25–35]. Besides the impulsive effects, time delays exist widely in many different areas of real world, which may cause oscillation, divergence, chaos, instability or other undesirable performances in the system. Recently, many efforts have been devoted to study the stability and asymptotic behavior of real-valued impulsive differential systems with delays, and many significant achievements have been reported [36–42]. Therefore, it is necessary and natural to introduce impulsive and delay effect into complex-valued differential systems. More recently, some interesting results on complex-valued impulsive differential systems have been reported. In [43], several stability conditions of complex-valued impulsive system were established by the Lyapunov function in the complex fields. In [44], the existence and uniqueness conditions of solutions of complex-valued nonlinear impulsive differential systems were acquired by fixed point theory. In [45,46], using the Lyapunov functional method and the matrix inequality technique, several sufficient conditions were obtained to guarantee the global exponential stability of impulsive complex-valued neural networks with delays. In [47], a class of complex linear time-varying impulsive systems was studied, and some sufficient and necessary conditions were derived for the state controllability and observability of the systems by the variation of parameters and the algebraic approach. However, to the best of our knowledge, there has been no result so far about the exponentially attractive set and the weak-invariant set of complex-valued nonlinear impulsive delay differential systems. Motivated by the above considerations, the objective of this paper is to investigate the exponentially attractive set and the weak-invariant set of complex-valued nonlinear impulsive delay differential systems by employing a combination of two inhomogeneous differential inequalities and the theory of matrix measure.

Our contributions consist of the following three parts.

- The introduction of the definitions of exponentially attractive set, weak-invariant set and weakness degree for the complex-valued impulsive delay differential system can be applied for the description of the asymptotic behavior of the complex-valued impulsive delay differential system.
- Two inhomogeneous impulsive delay differential inequalities are established which are important and useful for studying the asymptotic behavior of impulsive delay dynamical systems.
- We obtain the exponentially attractive set, weak-invariant set, and invariant set of the complex-valued nonlinear impulsive delay differential systems by combining the inhomogeneous impulsive delay differential inequalities and the matrix measure theory. Our results also hold when the complex-valued impulsive delay differential system degenerates to the complex-valued impulses-free delay differential system.

The rest of the paper is organized as follows. Section 2 introduces some notations and definitions. Section 3 develops two inhomogeneous impulsive differential inequalities. Based on the inhomogeneous differential inequalities, in combination with the matrix measure theory, we obtain the exponentially attractive set, weak-invariant set and invariant set in Section 4. Two examples that illustrate the main points of the paper are presented in Section 5. Section 6 draws conclusions.

2. Preliminaries

Throughout this paper, we adopt the following standard notations and definitions. Let E denote the n-dimensional unit matrix, $\mathbb{N} = \{1, 2, 3, \ldots\}$, $\mathbb{R}^+ = [0, \infty)$ and $i = \sqrt{-1}$. For $z \in \mathbb{C}^n$ and $A \in \mathbb{C}^{n\times n}$, let $\|z\|$ be any vector norm, x and y denote the real and the imaginary parts of z, respectively, and denote the induced matrix norm and the matrix measure, respectively, by

$$\|A\| = \sup_{z\neq 0} \frac{\|Az\|}{\|z\|},\ \ \mu(A) = \lim_{h\to 0^+} \frac{\|E + hA\| - 1}{h}.$$

$\mathcal{C}[\mathbb{X}, \mathbb{Y}]$ denotes the space of continuous mappings from the topological space $\mathbb{X}$ to the topological space $\mathbb{Y}$. $\mathcal{PC}[\mathbb{J}, \mathbb{R}^n]$ ($\mathcal{PC}[\mathbb{J}, \mathbb{C}^n]$)=$\{\psi : \mathbb{J} \to \mathbb{R}^n(\mathbb{C}^n)\,|\,\psi(s)$ is continuous for all but at most countable points $s \in \mathbb{J}$ and at these points $s \in \mathbb{J}$, $\psi(s^-)$ and $\psi(s^+)$ exist, and $\psi(s) = \psi(s^+)\,\}$ where $\mathbb{J} \subset \mathbb{R}$ is an interval, $\psi(s^-)$ and $\psi(s^+)$ denote the left-hand and right-hand limits of the function $\psi(s)$ at time s, respectively. Let $D^+\varphi(t)$ denote the upper right-hand derivative of $\varphi(t)$ at time t.

In this paper, we consider the following impulsive nonlinear complex-valued system with time-varying delays:

$$\begin{cases} \dot{z}(t) = A(t)z(t) + B(t)\bar{z}(t) + C(t)z(t-\tau(t)) + D(t)\bar{z}(t-\tau(t)) + f(t, z(t), z(t-\tau(t))) \\ \qquad\quad + g(t, \bar{z}(t), \bar{z}(t-\tau(t))),\ \ t \neq t_k, t \geq t_0, \\ \Delta z = z(t_k^+) - z(t_k^-) = H_k(t)z(t_k^-),\ \ k \in \mathbb{N}, \\ z(t_0 + s) = \phi(s), s \in [-\tau, 0], \end{cases} \tag{1}$$

where $z \in \mathbb{C}^n$ represents the state variable, $\bar{z} = x - iy$ is the conjugate of z, $\tau(t)$ is the time-varying delay function with $0 \leq \tau(t) \leq \tau$, τ is a positive constant, $A(t), B(t), C(t), D(t) \in \mathcal{C}[[t_0, \infty), \mathbb{C}^{n\times n}]$, $H_k \in \mathbb{C}^{n\times n}$, $g, f : [t_0, \infty) \times \mathbb{C}^n \times \mathbb{C}^n \to \mathbb{C}^n$ are complex-valued continuous functions, the initial function $\phi(s) = (\phi_1(s), \ldots, \phi_n(s))^T \in \mathcal{PC}[[-\tau, 0], \mathbb{C}^n]$, and the fixed impulsive moments $t_k (k \in \mathbb{N})$ satisfy $t_0 = 0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$ and $\lim_{k\to\infty} t_k = \infty$. Throughout this paper, we assume that for any $\phi(s) \in \mathcal{PC}[[-\tau, 0], \mathbb{C}^n]$ with the norm $\|\phi\|= \sup_{-\tau\leq s\leq 0} \|\phi(s)\| < \infty$, there exists at least one solution of (1), which is denoted by $z(t, t_0, \phi)$ (simply $z(t)$ if no confusion should occur). One may refer to [44] for the result on the existence and the uniqueness of solutions of complex-valued impulsive differential systems.