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Multistability of complex-valued neural networks with time-varying delays

Xiaofeng Chen[a], Zhenjiang Zhao[b], Qiankun Song[a,*], Jin Hu[a]

[a] Department of Mathematics, Chongqing Jiaotong University, Chongqing 400074, China
[b] Department of Mathematics, Huzhou Teachers College, Huzhou 313000, China

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ABSTRACT

In this paper, the multistability problem is studied for an n-dimensional delayed complex-valued neural networks with two general classes of activation functions. After splitting the state space to multiple subsets, based on the fixed point theorem, it is shown that such complex-valued neural networks can have 9^n equilibria, each of which is located in one of the subsets. Furthermore, some sufficient conditions are derived for the local exponential stability of some equilibria by employing the property of activation functions and inequality technique. As an application of these results, some criteria are obtained for checking the coexistence and exponential stability of multiple equilibria of real-valued neural networks. Two examples are performed to illustrate and validate the theoretical findings.

1. Introduction

Over the past several decades, owing to the fruitful applications in pattern recognition, associative memory, signal processing, image processing, combinatorial optimization, and other fields, neural networks have attracted a great deal of attention, and a large number of relevant results on their dynamical behaviors have been reported in the existing literature, see [1–12] and references therein. In the applications, especially, in physical systems dealing with electromagnetic, light, ultrasonic, and quantum waves, complex-valued neural networks become strongly desired. Therefore, a large number of complex-valued neural network models have been presented and studied, see [13–19] and references therein.

As is well known, compared with the real-valued neural networks (RVNNs), the states, connection weights and activation functions of the complex-valued ones are all complex-valued. Therefore, complex-valued networks (CVNNs) can be viewed as an extension of RVNNs. Apparently, CVNNs have different and more complicated properties than the real-valued ones. Hence, it is necessary and important to investigate the dynamical behaviors of CVNNs deeply. In recent years, considerable effort has been spent on developing stability theorem of CVNNs. In [20–31], the continuous-time CVNNs with delays were considered, and the boundedness, complete stability and exponential stability were investigated. In [32–35], the discrete-time CVNNs were investigated, and some conditions for the boundedness, global attractivity, complete stability, global exponential stability as well as global exponential periodicity of the considered neural networks were derived. In [36–38], the global stability was investigated for CVNNs on time scales, which is useful to unify the continuous-time and discrete-time CVNNs under the same framework.

* Corresponding author.
E-mail addresses: qiankunsong@163.com, qiankunsong@gmail.com (Q. Song).

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Generally speaking, the stability analysis for neural networks can be classified as monostability analysis and multistability analysis based on the numbers of stable patterns such as equilibria or limit cycles in the system. Mathematically, the notion of "monostability" means that the equilibrium of the networks exists and any state in the neighborhood converges to the equilibrium. The notion of "multistability" for a neural network is used to describe coexistence of multiple stable patterns. It is worth noting that the coexistence of multiple patterns is necessary in practical applications such as associative memory storage, pattern recognition, decision making, digital selection and analogy amplification. Recently, there has been an increasing interest in multistability analysis for neural networks [39–49]. In [39,40], the authors presented the existence of 2^n stable stationary solutions for a general n-dimensional delayed RVNNs with several classes of activation functions. In [41], the multistability has been developed for RVNNs with multilevel activations which are discontinuous but without delays. It is found that a k-neuron networks can have up to n^k locally exponentially stable equilibria, where n is the number of segments of the multilevel activation functions. In [42], for an n-neuron Cohen–Grossberg neural networks, the authors showed that there exist 2^n equilibrium points, which are locally exponentially stable and located in saturation regions. Besides some similar results were presented for ascertaining multiple periodic orbits when external inputs of the system are periodic. In [43], by formulating parameter conditions and using inequality technique, several delay-independent sufficient conditions ensuring the existence of 3^n equilibria and exponential stability of 2^n equilibria were derived for n-dimensional delayed competitive neural networks. In [44], the authors studied some multistability properties for a class of bidirectional associative memory recurrent neural networks with unsaturating piecewise linear transfer functions. In [45], the authors investigated the neural networks with a class of nondecreasing piecewise linear activation with $2r$ corner points, and the n-neuron dynamical systems can have and only have $(2r+1)^n$ equilibria under some conditions, of which $(r+1)^n$ are locally exponentially stable. In [46], the author investigated multistability of discrete-time Hopfield-type neural networks with distributed delays and impulses, by using Lyapunov functionals, stability theory and control by impulses. In [47], a set of sufficient conditions were presented to study the multistability, including the total number of equilibrium points, their locations, and stability for a class of neural networks with Mexican-hat-type activation functions. In [48], the problem of the coexistence and dynamical behaviors of multiple equilibria has been considered for competitive neural networks with a general class of discontinuous nonmonotonic piecewise linear activation functions. In [49], the multiplicity of almost periodic solutions was studied for a multidirectional associative memory neural networks with almost periodic coefficients and continuously distributed delays.

In most of the existing works, the multistability analysis has been mainly considered for RVNNs. To our best knowledge, the multistability for CVNNs has seldom been taken into account [50,51]. In [50], the authors considered the multistability problem of n-dimensional complex-valued recurrent neural networks with real-imaginary-type activation functions. Sufficient conditions were proposed for checking the existence of $[(2\alpha+1)(2\beta+1)]^2$ equilibria. Under these conditions, $[(\alpha+1)(\beta+1)]^2$ equilibria were locally exponentially stable and the others were unstable. In [51], the authors investigated the problem of multiple μ-stability of the CVNNs with unbounded time-varying delays. Some sufficient conditions were derived to ensure that existence of $[(\alpha+1)(\beta+1)]^2$ equilibria in which $[(\alpha+1)(\beta+1)]^2$ equilibria were locally μ-stable.

Strongly motivated by the above discussion and inspired by Huang et al. [50], in this paper, we investigate the problem of the coexistence and dynamical behaviors of multiple equilibria for an n-dimensional CVNNs with time-varying delays. Meanwhile, two general classes of activation functions are introduced into the CVNNs. The fixed point theorem and other analytical tools are used to develop some sufficient conditions that ensure that the CVNNs with time-varying delays can have 9^n equilibrium points, 4^n of which are locally exponentially stable. As a direct application of these results, we get some criteria on the coexistence and exponential stability of multiple equilibria of the RVNNs Finally, numerical simulations are presented to illustrate the effectiveness of the proposed theoretical results.

2. Problem description and preliminaries

We consider the following complex-valued neural networks with time-varying delays:

$$\dot{z}_i(t) = -\mu_i z_i(t) + \sum_{j=1}^{n} \alpha_{ij} f_j(z_j(t)) + \sum_{j=1}^{n} \beta_{ij} f_j(z_j(t-\tau_{ij}(t))) + J_i, \tag{1}$$

where $i = 1, 2, \ldots, n$; $z_i(t)$ represents the state of the ith neuron at time t; $\mu_i \in \mathbb{R}$ with $\mu_i > 0$; $\alpha_{ij}, \beta_{ij} \in \mathbb{C}$ denote the instantaneous feedback and delayed feedback connection strength from the jth to the ith unit; the time-dependent lags $\tau_{ij}(t) \geq 0$ are bounded continuous functions defined on $[t_0, +\infty)$, for some $t_0 \in \mathbb{R}$. $J_i \in \mathbb{C}$ correspond to the external bias; $f_j(\cdot) : \mathbb{C} \to \mathbb{C}$ are the activation functions.

We choose the following real-imaginary-type activation functions,

$$f_j(z) = f_j^R(\mathrm{Re}(z)) + \mathrm{i} f_j^I(\mathrm{Im}(z)),$$

where $f_j^R(\cdot)$ and $f_j^I(\cdot)$ are continuous functions from $\mathbb{R}$ to $\mathbb{R}$.