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Small-time expansions for state-dependent local jump–diffusion models with infinite jump activity

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Abstract

In this article, we consider a Markov process $\{X_t\}_{t\geqslant 0}$, which solves a stochastic differential equation driven by a Brownian motion and an independent pure jump component exhibiting both state-dependent jump intensity and infinite jump activity. A second order expansion is derived for the tail probability $\mathbb{P}[X_t \geqslant x + y]$ in small time t, where x is the initial value of the process and $y > 0$. As an application of this expansion and a suitable change of the underlying probability measure, a second order expansion, near expiration, for out-of-the-money European call option prices is obtained when the underlying stock price is modeled as the exponential of the jump–diffusion process $\{X_t\}_{t\geqslant 0}$ under the risk-neutral probability measure.

Keywords: Short-time asymptotics; Local jump–diffusion Markov models; Stochastic differential equations with jumps; Option pricing

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1. Introduction

In this work we consider a Markov process $X := \{X_t\}_{t\geqslant 0}$ with an infinitesimal generator of the form

$$Lf(x) = b(x)f'(x) + \frac{\sigma^2(x)}{2} f''(x) + \int_{\mathbb{R}_0} \big(f(x+\gamma(x,r)) - f(x) - \mathbf{1}_{\{|r|\leqslant 1\}}\gamma(x,r)f'(x)\big)\, \nu(x,r)dr, \tag{1.1}$$

where $\mathbb{R}_0 := \mathbb{R}\backslash\{0\}$ and $b : \mathbb{R} \to \mathbb{R}$, $\sigma : \mathbb{R} \to (0,\infty)$, $\gamma : \mathbb{R}\times\mathbb{R}_0 \to \mathbb{R}$, and $\nu : \mathbb{R}\times\mathbb{R}_0 \to (0,\infty)$ are deterministic functions satisfying appropriate conditions for the existence of such a process (see Section 2 for further details). Broadly, X can be constructed as the solution of a stochastic differential equation (SDE) of the form:

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t + dJ_t,$$

where $W := \{W_t\}_{t\geq 0}$ is a Wiener process and $J := \{J_t\}_{t\geq 0}$ is an independent pure-jump process, whose jump behavior is dictated by ν and γ as follows:

$$\mathbb{E}\big[\#\{s\in[t,t+\delta] : \Delta X_s \in (a,b)\}\big] = \mathbb{E}\big[\#\{s\in[t,t+\delta] : \Delta J_s \in (a,b)\}\big] = \mathbb{E}\left[\int_t^{t+\delta}\int \mathbf{1}_{\{\gamma(X_{s-},r)\in(a,b)\}}\nu(X_{s^-},r)drds\right], \tag{1.2}$$

for any $t\in(0,\infty)$, $\delta>0$, and $(a,b)\in\mathbb{R}\setminus\{0\}$. Intuitively, (1.2) tells us that the jump intensity of the process "near" time t depends on its state immediately before t via the function ν in that if $\nu(X_{t^-},r)$ is large (small), then we expect a higher (lower) intensity of jumps immediately after time t. In the particular case of $\gamma(x,r)\equiv r$, (1.2) reduces to

$$\mathbb{E}\big[\#\{s\in[t,t+\delta] : \Delta X_s \in (a,b)\}\big] = \mathbb{E}\left[\int_t^{t+\delta}\int_a^b \nu(X_{s^-},r)drds\right], \tag{1.3}$$

and $\nu(X_{t^-},r)$ has the usual interpretation of a stochastic jump intensity as defined in, e.g., [3] and [7]. That is, $\nu(x,r)$ measures the expected number of jumps, per unit time, with size near r when the process is at state x. State-dependent jump behavior as described above is an important feature that offers greater modeling flexibility to other commonly studied jump processes (see [17,13–15,8,9,21], and [26]). In the financial literature, for instance, [17] empirically asserts the plausibility of state-dependence not only on the drift and volatility of the returns but also on the jump intensity. Other areas of finance where state-dependent jump activity has been studied include interest and exchange rate modeling [8,13–15,26] and option pricing [10,21].

The generator (1.1) covers a wide range of processes. For a Lévy processes, b and σ are constants, $\gamma(x,r) = r$, and $\nu(x,r) = h(r)$, for a Lévy density $h : \mathbb{R}\setminus\{0\} \to [0,\infty)$ (i.e., $\int(x^2\wedge 1)h(x)dx < \infty$). When we simply have $\nu(x,r) = h(r)$, we recover the class of (local) jump–diffusion models studied in [12]. In that case, X can be constructed as

$$X_t = x + \int_0^t b(X_s)ds + \int_0^t \sigma(X_s)dW_s + \sum_{s\in(0,t]:|\Delta Z_u|\geq 1}\gamma(X_{s^-},\Delta Z_s) + \sum^c_{s\in(0,t]:0<|\Delta Z_u|\leq 1}\gamma(X_{s^-},\Delta Z_s), \tag{1.4}$$

where Z is a Lévy process with Lévy density h and $\sum^c$ denotes the compensated Poisson sum of the terms therein. The case of $\nu(x,r) = \lambda(x)p(r)$ with $\int p(r)dr = 1$ has been studied in [26].

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