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Abnormal shapes of production function: Model interpretations

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ABSTRACT

An abnormal non-monotonic shape of production function (response of obtained yield to increasing rates of mineral nitrogen fertilizers) has been observed in experimental field trials. Often, the observed effect (an inflection point, or intermediate plateau or even local undershoot of the “yield-fertilization” curve) is treated as a test distortion and will be ignored or sorted out. This article presents the authors' efforts to interpret and to explain similar phenomenon by means of investigating two mechanistic crop simulation models – AGROSIM and AGROTOOL. It is demonstrated that an imitation model can be used as a valuable tool of scientific research, allowing for the hypothesising of alternative understandings of non-trivial natural phenomena.

1. Introduction

The search for the correct mathematical formulation of the so-called “production function” has a long history, and is a well-known problem of theoretical agro-chemistry. The production function means the response of an actual or potential yield of agricultural crops to various environmental and management factors, in particular to different rates of mineral fertilizers. For many years the experimental determination of such dose-response relationships has been a subject of investigation in multivariate field tests. One related activity is to approximate observed experimental curves by simple functional dependencies (Griffin, 1987; Status and Methods, 1961). The background of this issue has a history of over 150 years and traces its roots back to classical research by Liebig (1855), Mitscherlich (1909). Table 1 presents a short summary of existing approximations of production functions.

However, in spite of the variety of proposed functional forms, they all only describe two principal shapes of a hypothetical response curve. The first one is a monotone increasing convex function (with or without saturation, i.e. characterised by limited or unlimited growth). The second is a unimodal function reaching its maximum at the optimal rate of fertilization and having a decreasing branch for super-optimal values of argument (negative impact of higher fertilization rates). Such a qualitative nature of the production function perfectly corresponds to the intuitive idea of the principal influence of a positive limiting factor on the production process of agricultural plants.

In fact, the relative efficiency of increasing doses of fertilizers (so called NUE – nitrogen use efficiency) must be the largest for small values, where a significant deficit of the limiting factor is seen. As fertilization doses increase, they lose their positive effect. Ultimately, very large doses can have a counterproductive influence on plant growth and development that leads to a decrease in the total yield.

Thus, typical shapes of production function (Curves 1 and 2 in Fig. 1) completely correspond to *a priori* understandings of plant reactions to possible excessive or lacking nutritional element.

At the same time, it is possible to find references to field as well as laboratory experiments which produce a more sophisticated shape of the production function curve (for wheat: Ivanova (1977); for ryegrass: Tumusiime et al. (2011); for barley: Surov et al. (1984), Emebiri et al. (2007); for rape: Seymour (2013); for cereals: Osmond et al. (2015); for nectarins: Daane et al. (1995)). In particular, this effect can sometimes be observed in test series with increasing doses of nitrogen fertilizers. The non-monotonic character of production function can be expressed by local decrease of relative NUE (inflection point), plateau-like segment or even a local minimum in the “yield-fertilization” response curve (Curve 3 in Fig. 1) appears in the medium interval of nitrogen fertilizer change.

Further increase of the nitrogen fertilizer dose leads to a return of the experimental production function to the “normal” shape. We hasten to point out that such a phenomenon is exhibited only in special, rarely occurring vegetation periods, i.e. for special combinations of environmental conditions such as abnormal early drought periods, high temperatures or other phenomena, and cannot be easily reproduced by field experiments. This in turn is often presented as an argument that the obtained results may be caused by methodological or experimental errors and, therefore, must be treated as merely test distortion. It seems, however, that the number of references to the same effect from

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Table 1
Approximations of the production function.

#	Approximation Y(X)	Author, year
1.	$Y = A{\cdot}X$, if $X < X_{max}$ $Y = Y_{max}$, if $X \geq X_{max}$	von Liebig (1855)
2.	$Y = A{\cdot}(1 - \exp(-k{\cdot}X))$	Mitscherlich (1909)
3.	$Y = a + b{\cdot}X - c{\cdot}X^2$	Pfeiffer and Fröhlich (1912)
4.	$Y = A{\cdot}\Pi(1 - \exp(-k_i{\cdot}X_i))$	Baule (1918)
5.	$Y = A - M{\cdot}R^X$	Spillman (1923)
6.	$Y = a{\cdot}X/(x + b)$	Briggs (1925) Rauterberg (1939)
7.	$Y = a{\cdot}X^{0.5}$	Boresch (1928)
8.	$Y = a{\cdot}X^b$	Sapehin (1923)
9.	$Y = a + b{\cdot}X - c{\cdot}X^n$	Bondorff (1924)
10.	$Y = a{\cdot}X^b{\cdot}\exp(-b{\cdot}z)$	Plessing (1943)
11.	$Y = a + b{\cdot}X + c{\cdot}X^2 + d{\cdot}X^3$	Stritzel (1958)
12.	$Y = A{\cdot}\exp(-z{\cdot}\log\|(X + 1)/(m + 1)\|^n)$	Boguslawski and Schneider (1962)
13.	$Y = A{\cdot}\log(X)$	Unknown author

independent researchers above-mentioned makes it a tendency which cannot quite simply neglected by the agricultural scientific community.

One example coming from the authors' own experience concern results of special field experiments with spring wheat performed at the Men'kovo Experimental Station of the Agrophysical Research Institute (St. Petersburg, Russia) in the 2012–2016 seasons of vegetation. They are presented below (see Table 2). The spring wheat cultivars "Esther" (2012) and "Darja" (2013–2016) were cultivated on sod-podzol sandy soil according to regional "good agricultural practice" for cereals production. Before sowing, nitrogen fertilizations varied from 0 to 180 kg N ha^{-1} at increments of 30 kg N ha^{-1}. Seven test sites in a quadruple repetition each (10 × 10 m) were randomly distributed at an experimental field with a good agricultural practice. It is seen that the production function in the experiment generally takes a typical shape (convex saturated or unimodal curve) in all seasons, whereas it contains an abnormal regions (local decrease of NUE) in 2013. We present two result datasets for 2013 which correspond to the experiments performed at two different agricultural fields (f1 – field with drainage system; f2 – field without drainage system). In Fig. 2 it is seen, that the production functions for both variants have well-expressed peculiarities (local plateau- or local minimum) near medium values of the argument (60 kg N ha^{-1} for field1 with drainage system and 90 kg N ha^{-1} for field2 without drainage system). Under the assumption that for the argument point of 90 kg ha^{-1} the observed value is absent for the "dashed" curve (field2) in Fig. 2, we can interpolate between the argument points 60 kg ha^{-1} and 120 kg ha^{-1} smoothly. The expected value will be approximately 4.2. The value observed in the experiment is 3.90 ± 0.16 (see Table 2). So, the expected value of 4.2 is out of confidence interval. Hence, the hypothesis of an existing plateau can be accepted. The same explanation can be used for the "dotted" curve (field1) in Fig. 2 for the argument point 60 kg ha^{-1} accordingly.

Unfortunately, we have no unambiguous and purely agronomic explanation of this effect at the moment. But the obtained results motivated us for investigation the observed case in more details. Indeed, sometimes similar results can be produced not in physical experiments, but in computer experiments, i.e. under the computation of eco-physiological mechanistic crop simulation models. As a result, a detailed investigation of all causal conditions and algorithms can offer a theoretical or model-based explanation for the phenomenon under consideration.

This article contains descriptions of computer-based investigations of abnormal production functions processed by means of two alternative crop simulation models. The first is AGROTOOL for spring wheat grown in 2013 at Men'kovo Experimental Station, Russia, and the second is AGROSIM for winter wheat grown in 1992 at Müncheberg Experimental Station, Germany, with extreme drought periods during spring, early summer and summer.

2. Material and methods

2.1. A description of the AGROTOOL crop model

AGROTOOL v. 3.5 is a generic crop model classified at the third production level according to de Wit's classification (de Wit, 1982). This means that the availability of water and nitrogen represents the main limiting factor in reducing potential photosynthesis-based productivity. The model consists of several independent, scalable and replaceable modules, interacting with each other at every time interval.

- ***The agrometeorological module*** is connected with a hydro-meteorological database that consists of all of the daily weather data required (minimum and maximum temperature, air humidity, precipitation and solar radiation characteristics).
- ***The module of solar radiation and photosynthesis*** calculates the daily sum of solar radiation intercepted and absorbed by plants, as well as the daily sum of accumulated assimilates due to photosynthesis and dark metabolism.

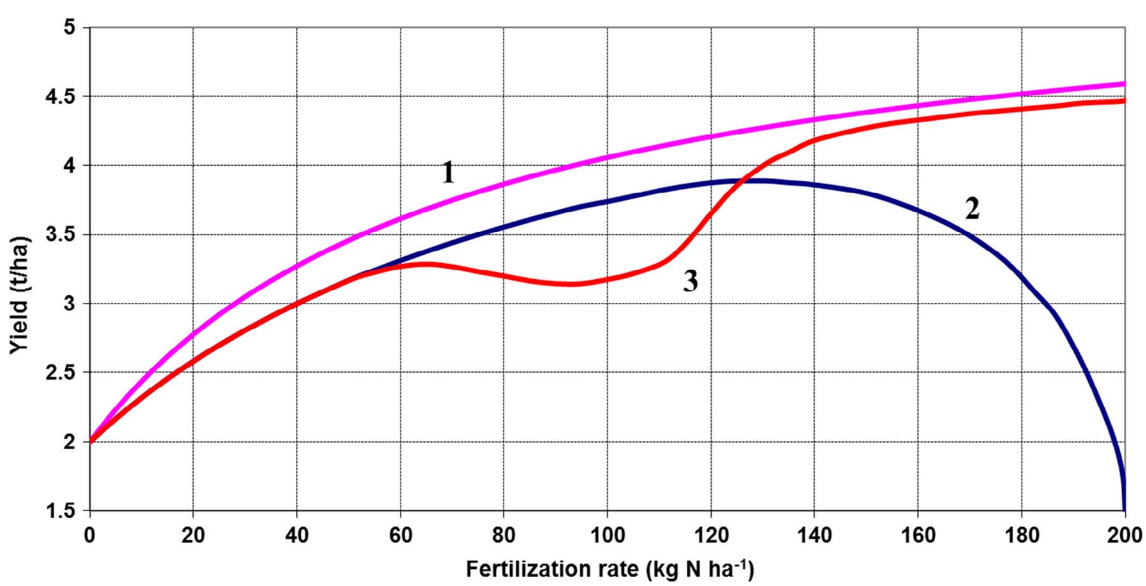

Fig. 1. "Typical" (Curves 1 & 2) and "abnormal" (Curve 3) shapes of "fertilization productivity of cereals" response curves.