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Combined cutting stock and lot-sizing problem with pattern setup

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A B S T R A C T

This paper considers a real-life problem that arises in a leading company in China involving the production of extra-high-voltage and high-voltage switch equipments. It concerns combining the one-dimensional cutting stock and lot-sizing problems. In addition to minimizing material waste, the number of cutting patterns required is also minimized. Such problem also occurs in other enterprises in paper, furniture and plastic film industries. We develop a mixed-integer linear programming model and propose a dynamic programming-based heuristic (DPH) to solve it. With a recursive formulation, each pattern is explored step by step with a global view. The computational results show that DPH is efficient, and yields results close to optimal solutions, where the average gap is 2.20% for small sized instances. The average cost gap is 4.19% compared with lower bound for medium or large sized instances. We also apply the proposed heuristic to real-life data to elaborate joint production and cutting plans and compare with the current procedure in practice. The total cost is reduced by 8.81% on average, which amounts to 3.46 million RMB in cost saving in 2016 for the investigated company. The average cost gap is 5.03% compared with the lower bound. The total cost is reduced by 3.67% on average compared with the two-stage independent decision-making method commonly applied in industry.

1. Introduction

Cutting stock is among the most extensively studied problems in operations research owing to its wide range of applications in the wood, steel, film, and leather cutting industries. In general, it concerns cutting a number of small pieces out of large objects to satisfy customer demands.

In this paper, we concentrate on a combined one-dimensional cutting stock and lot-sizing problem. This study is motivated by a real-life problem in a company in China that its workshop specializes in cutting metal (called as input rods) into tubes (called as output rods) of various lengths. The company collects orders in terms of tubes that specify tube length, quantities, and delivery time expressed in weeks. From these weekly orders, the company groups the orders together with the same delivery time and length.

In the production process, multiple identical input rods can be bundled together and cut simultaneously. For the scenario with automatically controlled machines, instead of being bundled physically, identical input rods can be successively put into the machine and cut in the same way. We say that these input rods are cut with the same cutting pattern, or simply pattern for short. The number of input rods cut with the same pattern is the number of occurrences of the corresponding pattern. For input rods cut with different patterns, operators should suspend the machine and adjust the cutting knives. According to Vanderbeck (2000), changing over from one cutting pattern to another involves significant setup times (for adjusting of knife positions) and costs (such as those associated with the waste incurred in trial runs). Therefore, switching between different patterns incurs a pattern setup cost. The company needs to decide whether production will occur, the production quantity for tubes of various length and the cutting patterns in each period of the planning horizon to satisfy orders while minimizing total cost, including production setup, pattern setup, inventory holding, and material waste costs.

Such problem also occurs in other enterprises including cutting process. For example, in paper industries, a typical problem consists of cutting large paper rolls into smaller rolls to satisfy the customer demands over a planning periods. In furniture industries, rectangular plates are cut in order to produce smaller pieces for the assembly of final furniture ordered by customers. In plastic film industries, a set of smaller rolls are cut to produce finished rolls. In these cutting process, changing over from one cutting pattern to another involves significant setup times and costs. In these manufacturing industries with cutting process, in elaborating the

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production and cutting plans, many companies have no optimization tools to aid decision making. They follow some routine procedures formed through individual experience. The company first determines the quantity of output rods produced in each period by solving the production planning problem. Then, they elaborate the corresponding cutting plan. From a global point of view, these two problems cannot be solved separately. Separate optimal solutions to these interdependent problems most likely result in a sub-optimal solution to the combined problem. An optimal solution involves the integration of the cutting stock and the lot-sizing problems.

Therefore, in this paper, we focus on a general problem: combined cutting stock and lot-sizing problem with pattern setup. This problem arises in the production system with features such as medium term planning horizon, deterministic demand, large production setup cost and negligible pattern setup cost. Related research in the literature can be summarized and classified into four categories:

- Cutting stock problem with pattern setup (CSPS)
  The classical cutting stock problem is widely acknowledged as NP-hard. It has been intensively studied in one or more dimensions. See Valério de Carvalho and Guimarães Rodrigues (1995); Dowsland and Dowsland (1992); Dyckhoff (1990); Ferreira and Oliveira (2008); Hinxman (1980); Liu et al. (2011); Lodi et al. (2002). Oliveira and Wäscher (2007) and Bennell et al. (2013b) reviewed recent developments in cutting stock problems.
  However, in industrial practice, switching between different cutting patterns in the cutting process can interrupt production and incur a setup cost. Therefore, the pattern setup cost is introduced into the classical cutting stock problem and considered as CSPS. In addition to minimizing the trim loss, the number of patterns needs to be minimized as well. Since these two objectives are in conflict, CSPS is generally formulated as a multi-objective model or a weighted single-objective model.
  Haessler (1975) was the pioneer researcher focusing on such a problem. He noted that switching in different patterns may incur additional setup cost, and the number of cutting patterns should be controlled. Chu and Antonio (1999) considered such a problem in one dimension and transformed the multi-objective function into a mono-criterion function through weighting. They proposed approximation algorithms that provided solutions very close to the optimum. Vanderbeck (2000) solved the pattern minimisation problem exactly by branch-and-price. Foerster and Wascher (2000) proposed an improving method to reduce the number of patterns. Yanasse and Limeira (2006) developed an efficient hybrid heuristic. Belov and Scheithauer (2007) proposed a sequential heuristic to combine the number of different cutting patterns and open stacks minimization. Alves et al. (2009) explored new lower bounds based on a different integer programming model for CSPS, and valid inequalities were proposed to strengthen the model. Araujo et al. (2014) proposed a genetic algorithm and obtained a set of Pareto optimal solutions. We also focused on CSPS in two dimensions with identical or variable-sized input rods (Liu et al., 2014; 2012).
- Multiperiod cutting stock problem (MCSP)
  MCSP arises when ordered output rods are required in different periods of a finite planning horizon. It is possible to reduce trim loss by producing some output rods in advance. This problem consists of determining whether the output rods is brought forward produced in each period and the corresponding cutting patterns in order to balance the increased inventory holding cost with the decreased material waste cost.
  Poldi and de Araujo (2016) developed two mathematical models for MCSP in one dimension. The objective function consisted of trim loss, and the inventory holding cost of output and input rods. A heuristic with a rolling horizon strategy was also designed. For the two-dimensional problem, Nonås and Thorstenson (2000) studied a continuous demand scenario. To simplify the problem, they assumed that cutting patterns were designed in advance and considered as inputs instead of decision variables. Arbib and Marinelli (2005) focused on such a problem to minimize the total trim loss cost over the planning horizon. Neither the pattern setup nor production setup cost was considered in this study. Poltroniere et al. (2008) investigated such a problem in paper production considering the width and type of paper as well as demand. They also considered pattern setup cost, and proposed two decomposition heuristics.
- Cutting stock problem with due dates (CSPD)
  CSPD is based on the classical cutting stock problem that considers the due dates of customer orders. It determines cutting sequences and patterns to fulfill customer orders and satisfy time constraints. In addition to minimizing trim loss, it seeks to minimize the completion time or tardiness in completing customer orders. In MCSP, products produced in advance are delivered in the stipulated period, which incurs inventory holding cost. In contrast to MCSP, in CSPD, customer orders are completed in time or before the due date; otherwise, a penalty cost is incurred for tardiness.
  Hendry et al. (1996) first studied such a problem in the copper industry. The objective was to minimize the number of input rods used as well as total production time. They proposed a decomposed approximate approach. Reinertsen and Vossen (2010) determined the sequence and combination of orders with the due dates. The objective was to minimize total tardiness cost. Bennell et al. (2013a) studied a two-dimensional bin packing problem with due dates. The objective is not only to minimize the number of bins, but also to minimize the maximum lateness of the rectangles. A genetic algorithm was proposed. Based on Arbib and Marinelli (2014); Reinertsen and Vossen (2010) provided a comprehensive study on cutting stock with due dates. They proposed an exact mathematical formulation and developed an effective heuristic. Arbib and Marinelli (2017) proposed a time-indexed Mixed Integer Linear Programming formulation for CSPD. The model was decomposed and solved by column generation procedure.
- Cutting stock with lot-sizing problem (CSLS)
  The lot-sizing problem determines the production quantities in each period over the planning horizon. The production setup cost is a fixed cost incurred if the production quantity is strictly positive. The problem aims to minimize total cost, including production cost, production setup cost, and inventory holding cost. Some research results can be seen in Bahl et al. (1987); Karimi et al. (2003); Maes et al. (1991); Zhong et al. (2015).
  CSLS combines cutting stock with the lot-sizing problem. It considers the time, quantity, and the corresponding occurrences of cutting patterns in order to produce output rods over the planning horizon, such that all demand is satisfied at minimum cost, including production setup, inventory holding, and trim loss costs. Gramani and França (2006) investigated the combined problem and designed an approximate method based on the network shortest-path problem. They assumed that the whole demand of the following periods could either be produced in advance or just in their required periods. Demand of output rods in a period could not be divided and produced in different periods.

Table 1summarizes existing related research. As mentioned above, the combined cutting stock and lot-sizing problem with pat-