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Valid inequalities for two-period relaxations of big-bucket lot-sizing problems: Zero setup case[☆]

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ABSTRACT

In this paper, we investigate two-period subproblems for big-bucket lot-sizing problems, which have shown a great potential for obtaining strong bounds. In particular, we investigate the special case of zero setup times and identify two important mixed integer sets representing relaxations of these subproblems. We analyze the polyhedral structure of these sets, deriving several families of valid inequalities and presenting their facet-defining conditions. We then extend these inequalities in a novel fashion to the original space of two-period subproblems, and also propose a new family of valid inequalities in the original space. In order to investigate the true strength of the proposed inequalities, we propose and implement exact separation algorithms, which are computationally tested over a broad range of test problems. In addition, we develop a heuristic framework for separation, in order to extend computational tests to larger instances. These computational experiments indicate the proposed inequalities can be indeed very effective improving lower bounds substantially.

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1. Introduction

The lot-sizing problem aims to determine an optimal production plan detailing how much to produce and stock in each time period of the planning horizon, given manufacturing system limitations such as machine capacities and customer orders/forecasted demand. Due to its strong impact on manufacturing companies' performance in terms of customer service quality and operating costs, lot-sizing has been a very active research area for many decades with significant attention from researchers as well as practitioners. Due to its practical importance and limited knowledge in the literature, we focus in this paper on the *multi-item* lot-sizing problem with *big-bucket* capacities, where each resource is shared by multiple items and more than one type of item can be produced in any time period. We study this problem from a theoretical perspective, where we analyze a two-period relaxation of this problem and characterize its important properties. Our main contributions are (i) several families of new valid inequalities and their

facet-defining properties for the relaxations of the two-period subproblem, (ii) novel extensions of these inequalities into the original space of the two-period relaxation, as well as new valid inequalities for the original space, and (iii) exact separation algorithms designed to test the practical strength of the proposed inequalities. We also develop a simple but effective heuristic approach in order to extend computational experiments. Our computational results show that the proposed inequalities have great potential to strengthen the lower bounds significantly.

1.1. Literature review

Most lot-sizing problems are inherently difficult problems: from the theoretical complexity perspective, even the multi-item problem with a single joint capacity and without setup times is strongly \mathcal{NP} -hard (Chen & Thizy, 1990). From a computational perspective, problems with multiple items and capacities, in particular of industrial scale, remain notoriously difficult to solve to optimality, often resulting in high duality gaps (Buschkühl, Sahling, Helber, & Tempelmeier, 2010). Therefore, there is a wide spectrum of research on lot-sizing problems, ranging from practically efficient heuristics (e.g., Federgruen, Meissner, & Tzur, 2007) and meta-heuristics (e.g., Jans & Degraeve, 2007) to mathematical programming techniques, which we discuss next in more detail due to their relevance to our study.

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Because of their complexity, most researchers in the mathematical programming community studied special cases of lot-sizing problems, which still provide valuable insights on some inherent structures of more general problems and hence support the solution methodologies proposed. The exact approaches most often employed either defining valid inequalities (e.g., Barany, Van Roy, & Wolsey, 1984; Küçükyavuz & Pochet, 2009; Pochet & Wolsey, 1994) or extended reformulations (e.g., Eppen & Martin, 1987; Krarup & Bilde, 1977; Pochet & Wolsey, 2010) for variants of single-item problem, some of which were also extended to multi-item problems (e.g., Belvaux & Wolsey, 2000). There are also few studies using other techniques such as Lagrangian relaxation (e.g., Billington, McClain, & Thomas, 1986) and Dantzig–Wolfe decomposition (e.g., Degraeve & Jans, 2007). Pochet and Wolsey (2006) provide a thorough review of different variants of lot-sizing problems, their complexities and solution methods used. Most recently, there have been insightful polyhedral results on multi-level problems, such as the valid inequalities of Zhang, Küçükyavuz, and Yaman (2012), and the compact formulations of Van Vyve, Wolsey, and Yaman (2014) for *small-bucket* capacities, i.e., items do not share resources.

Despite this extensive literature, the research explicitly investigating complications arising from multiple items competing for the limited capacities inherent in big-bucket problems is rather limited, and only few exceptions exist to the best of our knowledge. The polyhedral analysis of a single-period relaxation by Miller, Nemhauser, and Savelsbergh (2000, 2003) provided us some insightful properties of this polyhedron including new valid inequalities. The study of Jans and Degraeve (2004) presented various decompositions and indicated that period decompositions provide stronger bounds, which is recently investigated further by the branch-and-cut framework of de Araujo, Reyck, Degraeve, Fragkos, and Jans (2015) resulting in promising computational results with regards to gaps. The work of Van Vyve and Wolsey (2005) obtained strong lower bounds, most often stronger than any previous results, by applying approximate extended reformulations only for a small number of periods. The extensive computational study of Akartunali and Miller (2012) noted the bottleneck in big-bucket problems as the lack of a good understanding of the convex hull of single-machine, multi-period problems. This motivated the novel framework of Akartunali, Fragkos, Miller, and Wu (2016), where the smallest such problem, a two-period relaxation, is used to separate all violated inequalities by generating the extreme points of its convex hull, without pre-defining families of inequalities. The computational results of this study have shown great promise to significantly close duality gaps for big-bucket problems in general, which motivated us to study such a two-period relaxation from a polyhedral perspective.

In this paper, we present our work investigating the special case of zero setup times. This does not only enable us to analytically study inherent structures and hence provide useful insights that can be potentially extended to more complicated problems, but also improve our understanding about this multi-item problem with zero setup times that has been actively studied for many years in the lot-sizing literature, e.g., the earlier work of Dixon and Silver (1981) proposed heuristic approaches for this problem, and essentially the motivation for the seminal work of Padberg, van Roy, and Wolsey (1985) stemmed from this problem. From a practical perspective, it is worth noting that setup times are not necessarily zero, however, using zero setup times has been a very effective modelling approach in case of “negligible setup times”, i.e., very low setup times compared to processing times, in order to reduce the problem complexity, see, e.g., (Kuik, Salomon, & van Wassenhove, 1994) for a discussion. Negligible setup times can be observed in various manufacturing settings, e.g., assembly operations in automobile industry and packing operations in food industry. Moreover, as noted by Olaitan and Geraghty (2013)

technological advances such as agile tooling and material handling make it possible to produce different products on the same set of machines and therefore enable production line designers reduce setup costs significantly. We finally refer the interested reader to the review of Jans and Degraeve (2008) for a thorough discussion about this special case, achievements and open challenges.

In the next section, we present the problem formulation and the two-period relaxation X^{2PL} , originally proposed by Akartunali et al. (2016), and study some of its polyhedral properties, including the special case with no setup times. In Section 3, we present two relaxations of X^{2PL} , propose a number of valid inequalities for these relaxations and discuss their facet-defining properties. Section 4 presents novel extensions of these inequalities into the original space of X^{2PL} , as well as a new family of valid inequalities. We present exact separation algorithms in Section 5, and computationally test the strength of the inequalities in Section 6, which show promising results for their effectiveness. We also develop a simple but effective heuristic separation approach in Section 7 enabling experimentation with larger instances, where further encouraging results are obtained. We conclude the paper with a discussion of possible extensions and generalizations. We note that all essential proofs are provided in the Online Supplement due to their lengthy and involved nature.

2. A two-period relaxation for big-bucket lot-sizing problem

Before we define and study the two-period relaxation of interest, we first provide the mathematical formulation of the multi-item lot-sizing problem with big-bucket capacities. We let T , I and R denote the sets of *time periods*, *items*, and *machine (resource) types*, respectively. We represent the production, setup, and inventory variables for item i in period t by x_t^i , y_t^i , and s_t^i , respectively.

$$\min \sum_{t \in T} \sum_{i \in I} f_t^i y_t^i + \sum_{t \in T} \sum_{i \in I} h_t^i s_t^i \quad (1)$$

$$\text{s.t. } x_t^i + s_{t-1}^i - s_t^i = d_t^i \quad t \in T, i \in I \quad (2)$$

$$\sum_{i \in I} (a_r^i x_t^i + ST_r^i y_t^i) \leq C_t^r \quad t \in T, r \in R \quad (3)$$

$$x_t^i \leq M_t^i y_t^i \quad t \in T, i \in I \quad (4)$$

$$y \in \{0, 1\}^{|T| \times |I|}; x, s \geq 0 \quad (5)$$

The objective function (1) minimizes total cost, where f_t^i and h_t^i indicate the setup and inventory cost coefficients, respectively. The flow balance constraints (2) ensure that the demand for each item i in period t , denoted by d_t^i , is satisfied. We note that the model can be generalized to involve multiple levels (see, e.g., Akartunali & Miller, 2012), however, we omit this for the sake of simplicity. The big-bucket capacity constraints (3) ensure that the capacity C_t^r of machine r is not exceeded in time period t , where a_r^i and ST_r^i represent the per unit production time and setup time for item i , respectively. Constraints (4) guarantee that the setup variable is equal to 1 if production occurs, where M_t^i represents the maximum number of item i that can be produced in period t , which is the minimum of either the remaining cumulative demand or the capacity available. Finally, the integrality and non-negativity constraints are given by (5).

2.1. A two-period relaxation: X^{2PL}

Next, we present the feasible region of a two-period, single-machine relaxation, as originally proposed by

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