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Robust single-item lot-sizing problems with discrete-scenario lead time

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ABSTRACT

In this paper, we consider the minmax robust lot-sizing problem (RLS) with uncertain lead times. We explore the uncapacitated (RULS) and capacitated (RCLS) cases with and without lost sales, with discrete scenarios. We provide a complexity analysis proving that robust lot sizing problems are NP-hard even in the case with two scenarios. For each case, a mixed integer programming model is proposed. We show that several optimality conditions for the deterministic cases provided in Wagner and Whitin (1958), Aksen et al. (2003), as well as a classical facility location based model, are no longer valid. However, we generalize these properties to the lot sizing problem with deterministic but dynamic lead times and demonstrate their validity. Numerical experiments on instances with up to 400 periods and 25 scenarios reveal that, against all expectation, the capacitated version of the robust lot sizing problem is easier to solve than the uncapacitated version.

1. Introduction

The lot sizing problem decides when and how much to produce to satisfy the demand while minimizing the total costs. Since Wagner and Whitin (1958), a lot of research has been interested in this problem. For the single-item lot sizing problems, the readers can refer to the excellent review presented in Brahimi et al. (2006). The authors present four possible formulations used in literature. These formulations are the aggregate, disaggregate or facility location formulation, shortest path proposed by Evans (1985) and finally a formulation without inventory variables using the fact that the inventory can be expressed in function of the other variables of the model.

Most lot sizing problems are hard to solve. Some complexity results can be found in Florian and Klein (1971). They prove that the single item capacitated problem is NP-hard for quite general objective functions. Problems with concave cost functions and no capacity limits (Wagner and Whitin (1958) or constant capacities Florian and Klein (1971)) are solvable in polynomial time. Also lot sizing with convex cost functions and no setup cost is polynomially solvable. The deterministic ULS problem can be solved in $O(T \log T)$ according to Wagelmans et al. (1992). Akbalik and Rapine (2013) isolate some cases of ULS with time dependent fixed batch costs being NP-hard. Hellion et al. (2012), on the other hand, report a polynomial case for single item capacitated lot sizing problem with minimum order quantity and concave costs solved by a $O(T^5)$ time algorithm.

In many situations the assumption of known, deterministic demand and lead time is not necessarily realistic. The demand for each time period is not known in advance. The lead time of items can be uncertain for several reasons such as machine breakdowns, quality or transportation issues. Significant research is developed on optimal inventory policies by assuming that demands are independent and follow certain distributions. In Clark and Scarf (1960), the classical (s, S) policies are considered. An extension to demand uncertainty with a set of scenarios is presented in Guan and Miller (2008). A two-stage minimax RULS with interval uncertain demands is studied in Zhang (2011). Single-item single-period problems with uncertain lead times are solved by methods based on the well known Newsvendor model (Arrow et al. (1951)). The assembly system with multiple periods, constant demand and probabilistic lead times are investigated in Dolgui and Ould-Louly (2002), Louly et al. (2008). Louly and Dolgui (2013) propose a general analytical solution for any discrete probability distributions of lead time on a single item inventory with a fixed demand problem. Altendorfer (2015) extends this work by the stochastic demand occurrence and timing.

In this paper we address several extensions of the minmax robust lot sizing problem (RLS). RLS consists of planning the production of a single item with a known demand over a horizon of T periods. The lead time of an order made in a period t depends on the period and the scenario. Setup and production costs are computed as a function of the order date. The products are kept in the inventory with a holding cost between the delivery date and the consumption date of the demand. The objective is to determine a production planning to minimize the total setup and production costs as well as the maximum holding cost over all scenarios.

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This paper has several contributions. First, to the best of our knowledge, the robust lot sizing problem under lead time uncertainties is addressed for the first time. Second, we provide a complexity analysis showing that minmax robust ULS even in the case with only two scenarios is NP-hard. Since minmax robust ULS generalizes the versions with capacity constraint, and with and without lost sales, they are also NP-hard. As an obvious extension, we also demonstrate that the optimality condition of Wagner and Whitin (1958) is not valid for these problems. Aksen et al. (2003) give complementary optimality conditions for the case with lost sales and certain lead times. We bring a counterexample establishing the evidence that these conditions are not valid in the case with uncertain lead times under discrete scenarios. On the other hand, we generalize these properties to the lot sizing problem with deterministic but dynamic lead times, which corresponds to the case with only one scenario. Finally we extend our study to the capacitated cases.

2. Mathematical formulation of RULS without lost-sales

As mentioned above, several formulations are proposed for ULS and CLS problems in the literature. Through this section, we describe Mixed Integer Linear Models solving the minmax robust ULS and CLS.

2.1. The aggregate formulation of RULS problem

In the following, we present an aggregate formulation of the RULS problem. The notations are given below:

Parameters:

T	the total number of periods in the horizon
Ω	the set of scenarios
s_t	setup cost (fixed cost) of production (procurement) in period t
c_t	production unit cost at period t
h_t	unit inventory holding cost in period t
$Capa_t$	available capacity at period t
d_t	deterministic demand in period t
L_t^{ω}	random lead time for each scenario ω *in* Ω at each period $t \in T$

Variables:

In model (1)–(7), four different decision variables are used. X_t^{ω} and I_t^{ω} are the quantity of products delivered and inventory level at period t in scenario ω, respectively. Z_t is the quantity ordered at the period t and finally, Y_t is the binary variable taking the value 1, if an order is made at period t. It is worth of noting that the only binary variable of the model (Y_t) is independent of the scenarios.

The aggregate formulation (noted AG-RULS) is stated as follows:

$$\min \sum_{t=1}^{T} (s_t \cdot Y_t + c_t \cdot Z_t) + \max_{\omega \in \Omega} \left(\sum_{t=1}^{T} (h_t \cdot I_t^{\omega}) \right) \tag{1}$$

$$I_{t-1}^{\omega} + X_t^{\omega} = d_t + I_t^{\omega} \quad \forall\, t, \quad \forall\, \omega \tag{2}$$

$$Z_t \le M \cdot Y_t \quad \forall\, t \tag{3}$$

$$X_{t'}^{\omega} = \begin{cases} \sum_{t \le t'} Z_t, & \text{if } t + L_t^{\omega} = t' \\ 0 & \text{otherwise} \end{cases} \quad \forall\, t', \quad \forall\, \omega \tag{4}$$

$$Y_t \in \{0, 1\} \quad \forall\, t \tag{5}$$

$$Z_t \ge 0 \quad \forall\, t \tag{6}$$

$$X_t^{\omega}, I_t^{\omega} \ge 0 \quad \forall\, t, \quad \forall\, \omega \tag{7}$$

The objective function (1) minimizes the total setup and production costs as well as the maximum of the inventory holding costs over all scenarios. Constraints (2) are the inventory balance equations. Constraints (3) relate the binary setup variables Y_t to the continuous order production variables Z_t. The relation between the ordered and delivered quantities is defined by the constraints (4). Therefore, depending on the lead time, the scenario is that an order placed at period t will be received at $t' = t + L_t^{\omega}$. We underline that several orders made at different dates can be received at the same time. That is why, the received quantity is the sum of the orders having the same delivery date. Constraints ((5), (6) and (7)) give the definition domains of the variables.

2.2. The complexity of the RLS problem

Zhang (2011) reports that a RLS problem is NP-hard when the uncertainty is on the demand. Several robust problems are proven to be NP-hard even with two scenarios in Kouvelis and Yu (1996). In the following, we show that a robust uncapacitated lot-sizing problem with uncertain lead times and discrete scenarios is NP-hard even in the case with two scenarios.

Theorem 1. *A robust single-item uncapacitated lot - sizing problem with discrete-scenario lead times is NP - hard even in the case* $|\Omega| = 2$.

Proof. We will use the following polynomial transformation from a 3 dimensional matching problem which is known to be NP-complete (Karp (1996)).

Let J be a time interval where all the deterministic demand occurs, and I be the time interval in which, if an order is placed, it is received in J. In other words if $j_{\omega}(i) \in J$ where $j_{\omega}(i) = i + L_i^{\omega} \;\; \forall\, \omega \in \Omega = \{1, 2\}$, then $i \in I$. Let the demand of any period in J be equal ($d_j = d, \;\; \forall\, j \in J$), unit holding cost for all periods in J be H, and all setup and production costs be 0 $\forall\, i \in I$.

Suppose $J_1 = J_2 = J$, and a triple($\mu = (i, j_1(i), j_2(i))$) is the period i and two periods that a quantity ordered in i can be received following scenario 1 or 2, respectively. For a triple $\mu = (i, j_1(i), j_2(i)) \in I \times J_1 \times J_2$, $i \in I$, $j_1(i) \in J_1$ and $j_2(i) \in J_2$.

There is a solution to the robust single-item uncapacitated lot-sizing problem with zero cost, if and only if there exists a 3-dimensional matching $M \subset I \times J_1 \times J_2$ with $|M| = |J|$.

1. It is obvious that if there is a 3-dimensional matching M such that $|M| = |J|$, there is a solution to the robust single-item uncapacitated lot-sizing problem with zero cost. According to the definition of 3-dimensional matching, if there are two triples μ and μ' in M such that $\mu = (i, j_1(i), j_2(i))$, $\mu' = (i', j'_1(i), j'_2(i))$ and $\mu \ne \mu'$, then $i \ne i'$, $j_1(i) \ne j'_1(i)$ and $j_2(i) \ne j'_2(i)$. That implies
$$\bigcup_{\mu=(i,j_1(i),j_2(i)) \in M} j_1(i) = \bigcup_{\mu=(i,j_1(i),j_2(i)) \in M} j_2(i) = J$$
For all triple $\mu = (i, j_1(i), j_2(i)) \in M$, it is enough to order quantity d at dates i which will be received either at $j_1(i)$ or $j_2(i)$, following the scenario. In any scenario, all the demand in J is covered by the just in time deliveries. Therefore, for any scenario, the sum of all setup, production and holding costs will be zero.
2. If there is no a 3-dimensional matching M such that $|M| = |J|$, then $|M| < |J|$. Let $J_1^M \subset J$ and $J_2^M \subset J$ be subsets of J_1 and J_2 induced by M. We have
$$|J_1^M| \le J_1 \quad \text{and} \quad |J_2^M| \le J_2$$
with at least one of the inequalities being strict. For the sake of simplicity, let $|J_1^M| < J_1$. In that case there exists at least one $\tilde{j}$ such that $\tilde{j} \in J_1$ but $\tilde{j} \notin J_1^M$. In a feasible robust single-item uncapacitated lot-sizing solution, an order should be placed at date $\tilde{i} \in I$ such that $\tilde{i} + L_{\tilde{i}}^1 \le \tilde{j}$ and $\tilde{i} + L_{\tilde{i}}^2 \le \tilde{j}$ with at least one of the inequalities being strict. The cost triggered by this order is
$$\Delta_{\tilde{j}} = \max_{1,2} \{H \times (\tilde{j} - (\tilde{i} + L_{\tilde{i}}^1)), H \times (\tilde{j} - (\tilde{i} + L_{\tilde{i}}^2))\}$$
As $\min \Delta_{\tilde{j}} > 0$, the cost of any feasible solution to the Robust single-