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# Dynamic lot-sizing models for retailers with online channels

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ABSTRACT

This paper studies inventory replenishment planning problems for retailers with online channels. Such retailer is able to obtain advance demand information (ADI) in an environment of time-varying demands. We incorporate ADI into dynamic lot-sizing models to formulate the replenishment planning problems for retailers in three scenarios: (1) companies act as pure-play online retailers with customers homogeneous in demand lead time, (2) online customers are heterogeneous in demand lead time with priorities, and (3) retailers operate in a *bricks-and-clicks* structure, in which demands come from online and offline channels, with either independent or interactive channels. We formulate the problem in the general scenario of interactive demand channels as a mixed-integer linear programming model, and then develop a unified model through reformulation. Based on the optimality properties, we design a dynamic programming algorithm with polynomial running time to solve the unified model. The numerical experiments for several online retailers find that the method can significantly reduce the total inventory cost.

## 1. Introduction

Retailers have been expanding business with the use of online channels, and have generated the online retailing industry, which is playing an important role in today's economy (see Bell et al., 2012; Gupta et al., 2009). From 2014 on, the online retail sales in Europe will grow at a compound annual growth rate of 12% to reach 233.9 billion euros by 2018, and the total online retail spending in China will grow at a compound annual rate of nearly 20% to reach more than 1 trillion dollars by 2019 (Beeson et al., 2014; Zeng et al., 2015). Behind the glitter is that the competition on e-commerce market becomes fiercer than ever, since most retailers run their online business on thin margins (Asdemir et al., 2009). Online retailers are thereby devoted to reducing the after-sales operational costs (see, e.g., Agatz et al., 2008; Gong and de Koster, 2008). Optimizing inventory and fulfillment functions is one of such solutions (Randall et al., 2006).

Our research is motivated by inventory replenishment problems arising in a retailer selling electronic products online in Europe. Many commodities this retailer sells have a common feature of dynamic demands. For example, the demands of a new house cleaning product *IRobot* (see http://www.irobot.com/) are low during the period of initial sale, but keep increasing since more people get to hear about and accept it. Besides, under the contract of partnership with the corresponding manufacturer, this retailer enjoys a timely and sufficient supply of this very product. In such a setting, the manager needs to design an appropriate inventory replenishment policy to satisfy the demands at minimal costs. Other online retailers, including customized clothes sellers, assembled computers providers, and jewelers, can encounter similar problems (Chen et al., 2011). They can either manufacture products by themselves or resort to suppliers to feed time-varying demands. This study deals with inventory replenishment problems for these retailers with online business.

It is convenient for retailers with online channels to obtain advance demand information (ADI). Hariharan and Zipkin (1995) first identify the value of the customers' advanced warnings about orders in reducing inventory costs and define the time from a customer's ordering until the due date as *demand lead time*. Then Gallego and Özer (2001) officially coin the term of ADI, and define it as the resulting information when customers with different demand lead times place orders in advance of their needs. Retailers with online channels obtain ADI when customers place orders online, since these demands will usually be satisfied at a future delivery date. They may develop several strategies to make more ADI available. For example, some online retailers (e.g., Amazon) offer free shipment to encourage customers to wait, then have more time to handle the orders. Others (e.g., Apple and Xiaomi) can obtain ADI through offering preorder strategy, referring to

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the practice of a seller accepting orders before a product is released (Li and Zhang, 2013). In addition, online retailers can trace and analyze online clicking behavior of visitors through techniques like data mining and machine learning to estimate the converted demands in the future, which represent another type of ADI (Huang and Van Mieghem, 2014). Thereby, our research question is "how can we use ADI to develop an inventory replenishment policy for retailers with online channels to satisfy time-varying demands so that the total operational cost is minimized? "

As many retailers have implemented an ERP system integrated with a modular of dynamic lot-sizing (DLS) to manage the replenishment processes, the DLS model arises as a fundamental method in solving practical large-scale replenishment planning problems (Graves, 1982). To answer the aforementioned question in this setting, we model the inventory replenishment plans as uncapacitated single-item lot-sizing problems with consideration of ADI and flexible delivery. Retailers have developed new operational modes with the use of online channels. For example, traditional brick-and-mortar retailers have extended their Internet channels to bring the so-called "*bricks-and-clicks*" mode, and "pure-play" online retailers are becoming increasingly innovative by setting their own kiosks inside stores of well-known traditional offline retailers (Bell et al., 2012). We consider the replenishment problems in three retailing scenarios with online channels. In the first scenario, we consider companies as pure-play online retailers with customers homogeneous in demand lead time (i.e., all orders are sent out with the same delivery service). Then we extend it to the second scenario where online customers are heterogeneous in demand lead time with priorities, offering express and standard delivery options (e.g., Amazon, Boulanger (http://www.boulanger.com/), and Darty (http://www.darty.com/)). In the third scenario, retailers operate in a *bricks-and-clicks* structure, in which orders come from both online and offline demand channels. For instance, Apple runs both online and offline stores. The demand from those customers who directly visit an offline store to buy something is an offline demand channel. These channels can be either independent or interactive. The latter operational pattern provides customers with an option to place orders online and pick products in an offline store, called "buy-online, pick-up-in-store" (BOPS) by Gallino and Moreno (2014).

Based on the characteristics of these three scenarios, we specify the inventory replenishment process for each. We then identify the correlations among them, and show that the scenario of interactive demand channels is a generalization of all the scenarios considered in this study. As a result, we formulate the problem in this general scenario as a mixed-integer linear programming model. Furthermore, through reformulation, all the problems in different scenarios can be analyzed in a unified dynamic lot-sizing model by classifying different demand categories according to the corresponding delivery time windows. Based on the optimality properties, we design a dynamic programming algorithm with polynomial running time to solve the unified model. The numerical experiments for several online retailers in different settings show that the method can significantly reduce the total inventory cost.

The remainder of the paper is organized as follows: In the next section, we discuss the relevant literature. In Section 3, we depict the inventory replenishment processes in three retailing scenarios with online channels and use dynamic lot-sizing models with ADI and flexible delivery to formulate the research problems. In Section 4, we show that all the problems can be reformulated into a unified model, analyze the optimality properties for this model, and develop a polynomial time algorithm to solve it. Section 5 validates and verifies the models with numerical experiments based on selected online retailers. We make concluding remarks in Section 6.

## 2. Literature

Our study is of relevance to three research streams: dynamic lot-sizing models, the research of using ADI in inventory management, and the inventory policy of online retailing.

Dynamic lot-sizing problems have been vastly studied since Wagner and Whitin (1958). Researchers have been extending the lot-sizing problems in various settings. We can refer to several reviews on models category, algorithms, and demand patterns (e.g., Bahl et al., 1987; Karimi et al., 2003; Brahimi et al., 2006; Jans and Degraeve, 2007). Most lot-sizing models solve production or inventory planning problems in an environment of manufacturing, especially in MRP (material requirement planning) systems (De Bodt et al., 1984). Our research follows the stream by applying an uncapacitated (no restrictions on procurement and storage) DLS model to deal with the inventory replenishment problem for retailers with online channels under three different scenarios, which has seldom been considered before. We propose a general model that can be adopted to fit any one of the three retailing scenarios that involve online channels, and design an algorithm with polynomial running time to solve it.

Our research is also relevant to the use of ADI in inventory management (for a review, see Özer, 2011). ADI usually includes two parts: the observed part, referring to the known demands, which have been observed before the current period and will prevail in a future period; and the unobserved part, referring to those unknown and have not been observed yet (Gallego and Özer, 2001). For example, if a vector $D_t = (D_{t,t}, \ldots, D_{t,t+N})$ represents orders placed during period $t$ for period $s \in \{t, \ldots, t+N\}$ ($N$ is the information horizon), the observed part of ADI (known to us) at the beginning of period $t$ is $O_{t,s} = \sum_{r=s-N}^{t-1} D_{r,s}$, and the unobserved part (not yet known) is $U_{t,s} = \sum_{r=t}^{s} D_{r,s}$. ADI can also be perfect (i.e., future demands that are available prior to their materialization) or imperfect, referring to the uncertain indication of future orders (Tan et al., 2007). For instance, orders that are placed in advance without any change consititute "perfect ADI", and the number of customers interested in buying a product in a future date provides "imperfect ADI". Hariharan and Zipkin (1995) first study an inventory scenario where customers provide advance warning of their demands (i.e., ADI), and prove that the demand lead time shows the same effect as the reduction of supply lead time on the inventory system. Increasing demand lead time is more beneficial than reducing supply lead time when considering ADI with flexible delivery (Wang and Toktay, 2008). With the development of big data, recently scholars have begun to investigate how to incorporate online clickstreams data as ADI into classic inventory management (Huang and Van Mieghem, 2014). While the researches above use ADI in stochastic inventory systems, this study incorporates the observed part of ADI into dynamic lot-sizing models as inventory policies for retailers with online channels. Our research is the first to introduce the concept of ADI into dynamic lot-sizing models and identify the correlation between ADI and demand time window through reformulation.

The literature on inventory policy of online retailing mainly considers two kinds of online retailers, pure-play online retailers and retailers operating in a multi-channel or dual-channel environment. Several studies focus on inventory management of pure-play online retailers, for example, extending the use of a traditional two-stage serial inventory system in a setting of online retailing (Allgor et al., 2004), analyzing the strategy of maintaining an online retailer's own inventory (Chen et al., 2005), and designing an inventory allocation policy (Xu, 2005) or replenishment policy (Acimovic, 2012) to minimize the transportation costs or lower outbound shipping costs. More researches consider a multi-channel or dual-channel environment, i.e., companies run business both in a brick-and-mortar channel and an online channel (see, for instance, Cattani et al., 2006; Rodríguez and Aydın, 2015). With an integration of the direct and indirect channels by using the excess stock of retailer partners to fulfill online orders, online retailers can find the optimal parameters of supply contracts (Seifert et al., 2006). Some researchers suggest that retailers in a *bricks-and-clicks* structure can use the traditional inventory policy for stock