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Exact probability distribution function for the volatility of cumulative production

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In this paper we study the volatility and its probability distribution function for the cumulative production based on the experience curve hypothesis. This work presents a generalization of the study of volatility in [1], which addressed the effects of normally distributed noise in the production process. Due to its wide applicability in industrial and technological activities we present here the mathematical foundation for an arbitrary distribution function of the process, which we expect will pave the future research on forecasting of the production process.

INTRODUCTION

Understanding the volatile behaviour of industrial activities and the complexity related to them, intrigues many researchers. One of the stylized facts for describing this phenomenon is a well known experience curve. The concept of experience curves and empirical evidence for them were presented in Wright's [2] seminal paper, in which he first discovered the relationship of cost and quantity. Wright's curve is known in the literature as "learning curve", as it is based on "the more learning by more producing hypothesis", for describing the price-experience relationship. Wright realized that empirically the reduction of cost followed a constant proportion rate, as the production duplicated. In particular, the higher the experience in producing a specific product is, the lower its costs are, when the inflation is factored out.

The motivation of this paper comes from the fact that the experience curves hypothesis can provide a significant understanding of the market strategy, for instance export potentials due to the knowledge of experience levels, the prediction of future prices, given some information about the market costs decrease by some consistent rate of decline, the applicability in risks management, etc. (Note that the notion is suitable for cost control or forecasting over long range strategic development). Moreover the experience curve effect can be observed in any business, any industry, and any cost element [3].

The phenomenon depends on some crucial factors, i.e. competent management, technological improvement, etc. Furthermore there must be a characteristic pattern that causes this phenomenon, for instance a better development of better tools, automatization, training programs [4–7], prior experience and the work complexity task [8, 9].

The notion of experience curve could also describe the effect between business competitors, for example, who is faster by reducing the costs, which is an example of complex systems interactions and network.

There is a vast literature on empirical information about the experience curves, including a wide range of industrial activities, see e.g. [10]. The aforementioned work had a major impact on the development of this concept, by arguing that technical learning was a result of experience gained, based on the idea learning by doing. Some researchers question its usefulness for forecasting and planning the deployment of industrial and technological activities [1, 11–15]. In the aforementioned literature it has been found that experience curves can be used to estimate future technology costs, considering the shape of the forecast error distribution. (Note the finding depends on some parameters, for instance the length and the period of observed time series).

Despite the wide variety of empirical evidence of the experience curves, there is a lack of theoretical and mathematical framework of the concept. Motivated by this fact and by the fact that there exists a large number of cases where the distribution describing a complex phenomenon is not Gaussian, e.g. the price fluctuations of most financial assets [16], in this paper we present a theoretical, mathematical framework for describing a probability distribution function of the volatility of the cumulative production for an *arbitrary* probability distribution of noise. In analogy to the concept of learning curves, which is a relation between the input and the output of a learning process, one of our main findings shows the relation between previous and next probability distribution functions which characterizes their volatility.

Knowing such an important quantity, can describe the marketing and movement of products. It will allow us to understand the complex behaviour of the system and to calculate the various quantities, such as mean, variance and also higher order moments, price volatility correlation, etc. The correlation coefficients and moments are among the useful quantities to describe the dynamics and the correlations between the random variables. However, a precise investigation can only be achieved if the probability distribution function (PDF) of the variable is known. The derivation of this for production processes is the main objective of the present paper.