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Covering location problem of emergency service facilities in an uncertain environment

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ABSTRACT

In practical location problems on networks, the response time between any pair of vertices and the demands of vertices are usually indeterminate. This paper employs uncertainty theory to address the location problem of emergency service facilities under uncertainty. We first model the location set covering problem in an uncertain environment, which is called the uncertain location set covering model. Using the inverse uncertainty distribution, the uncertain location set covering model can be transformed into an equivalent deterministic location model. Based on this equivalence relation, the uncertain location set covering model can be solved. Second, the maximal covering location problem is investigated in an uncertain environment. This paper first studies the uncertainty distribution of the covered demand that is associated with the covering constraint confidence level α. In addition, we model the maximal covering location problem in an uncertain environment using different modelling ideas, namely, the (α, β)-maximal covering location model and the α-chance maximal covering location model. It is also proved that the (α, β)-maximal covering location model can be transformed into an equivalent deterministic location model, and then, it can be solved. We also point out that there exists an equivalence relation between the (α, β)-maximal covering location model and the α-chance maximal covering location model, which leads to a method for solving the α-chance maximal covering location model. Finally, the ideas of uncertain models are illustrated by a case study.

1. Introduction

As a valuable method for siting service facilities, facility locating is widely used in real life such as for emergency service systems. In recent decades, many researchers have studied the facility location problem [5,13,16]. In the event of an emergency, such as a terrorist attack (e.g., the events of September 11th), a major natural disaster (e.g., hurricanes), or common emergency situations (e.g., house fires and regular health care needs), the residents of an impacted area need to be moved to safe locations as quickly as possible, leading to a sudden and tremendous surge of demands for emergency services. Emergency service providers, such as fire departments and ambulance systems, must provide a high service level to ensure public safety. These services are often provided by vehicles based at fixed locations. As a critical part of emergency

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management, the emergency service facility location problem attempts to determine the best locations for facilities so that certain service-level objectives are optimized.

To determine the best locations for facilities on a network, several different models have been defined for different situations. One class of such models is that of set covering models, which are the most widespread location models for formulating emergency facility location problems. The first models of this type are models that address the location set covering problem (LSCP), whose aim is to determine the minimum number of service facilities that are required to cover all demand vertices. Since being introduced by Toregas et al. [27], the LSCP has become a very well-studied in location problem. Another important extension of the set covering model of emergency facilities is often called the maximal covering location problem (MCLP), which was first produced by Church and ReVelle [6]. In a standard MCLP, one seeks the location of a number of facilities on a network such that the covered demand is maximized. ReVelle et al. [25] applied heuristic concentration to the problem of large instances of the MCLP with high coverage percentages. Zarandi et al. [34] presented a customized genetic algorithm to solve MCLP instances with up to 2500 vertices. Yin and Mu [33] proposed a modular capacitated maximal covering location problem to allow for several possible capacity levels for the facility at each potential site.

In the literature, both the LSCP and MCLP have been studied in a deterministic framework in which the parameters are supposed to be positive crisp values. As we know, the highly unpredictable nature of emergencies may lead to indeterminacy in terms of both demands and response times. Some researchers believed such indeterminacy behaves like randomness. Based on this assumption, probability theory has been introduced to emergency service covering models to capture the indeterminacy of these problems. Daskin [7] formulated the maximum expected covering location problem to place p facilities on a network with the objective of maximizing the expected value of demand coverage. Berman et al. [4] studied the gradual covering location problem on a network with random demands. In recent years, many models have been extended to address other emergency service location problems by Batta et al. [1], Jia et al. [15], and Toro-Díaz et al. [28]. For additional research on location problems in indeterminate environments, we refer to Beraldi and Bruni [3], Wei et al. [31], and Du et al. [9].

It is undeniable that probability theory is a useful tool to deal with random factors. However, a fundamental premise of applying probability theory is that we should obtain a probability distribution close enough to the real frequency through statistics. In emergency circumstances, we frequently lack observation data. Usually, we have no choice but to invite some domain experts to evaluate the belief degree that each event will occur. Liu [23] noted that if we deal with belief degrees by probability theory, this may lead to counterintuitive results. How do we address belief degrees? To model this type of human uncertainty, an uncertainty theory was founded by Liu [19] and subsequently studied by many scholars. So far, uncertainty theory has become a branch of axiomatic mathematics for modelling human uncertainty. In this paper, we deal with the location problem of emergency service facilities just under this uncertainty.

As a type of mathematical programming involving uncertain variables, uncertain programming was first proposed by Liu [21] in 2009. After that, uncertain programming was widely applied in engineering, management and design. For instance, Gao [10] investigated the shortest path problem in an uncertain network. Zhang and Peng [35] investigated the Chinese postman problem in an uncertain environment, and they proposed three models according to different modelling ideas. Sheng and Yao [26] presented an uncertain transportation model, in which the costs, supplies and demands were assumed to be uncertain variables. Furthermore, uncertain programming was also applied to portfolio selection [2], the project scheduling problem [37], the price discrimination problem [29], the assignment problem [36], the newsboy problem [8], and the capital budgeting problem [17]. In particular, Gao [11] proposed the concepts of satisfaction degrees for both vertices and the whole network and then introduced two uncertain programming models for the single facility location problem. Zhou et al. [38] proposed a multi-objective uncertain model for the fire station location problem. Wang and Yang [30] studied a hierarchical facility location for reverse logistics in an uncertain environment. Wen et al. [32] investigated a facility location-allocation problem with the objective of minimizing the total transportation cost. Huang and Di [14] described the estimations of customers' positions by uncertain variables and investigated an uncapacitated facility location problem in an uncertain environment. Gao and Qin [12] studied a hub location problem by considering the travel times as uncertain variables and presented a chance-constrained programming model.

The facility location problem and the resource scheduling problem represent two critical parts of an emergency response network. Recently, Li et al. [18] investigated the problem of dispatching medical supplies in emergency events under the framework of uncertainty theory. In this paper, we will consider the location problems of emergency service facilities in an uncertain environment. Because of the highly unpredictable nature of both emergencies and the severity of an accident, the response times and the demands cannot be known precisely but can be regarded as uncertain variables. Based on such assumptions, we propose using uncertain programming in modelling the problems. This paper makes the following significant contributions to the literature. We model LSCP and MCLP in an uncertain environment. Three uncertain location models, i.e., the uncertain location set covering model, the (α, β)-maximal covering location model and the α-chance maximal covering location model, are formulated using different criteria. Within the framework of uncertainty theory, the crisp equivalents of the models are discussed.

The remainder of the paper is organized as follows. In Section 2, the location set covering problem in an uncertain environment is studied, and the equivalent deterministic location model is discussed. In Section 3, two types of uncertain maximal covering location models are proposed, and the distribution function of the total covered demand is discussed. In addition, the properties of the uncertain models are investigated, and then, the corresponding algorithms for solving the