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A shelf-space optimization model when demand is stochastic and space-elastic ☆

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ABSTRACT

The more customer demand is impulse-driven, the more it is space-dependent and the more it is subject to variation. We investigate the corresponding problem of retail shelf-space planning when demand is stochastic and sensitive to the number and position of facings. We develop a model to maximize a retailer's profit by selecting the number of facings and their shelf position under the assumption of limited space. The model is particularly applicable to promotional or temporary products.

We develop the first optimization model and solution approach that takes stochastic demand into account, since the current literature applies deterministic models for shelf-space planning. By the means of an innovative modeling approach for the case with space- and positioning effects and the conversion of our problem into a mixed-integer problem, we obtain optimal results within very short run times for large-scale instances relevant in practice. Furthermore, we develop a solution approach to account for cross-space elasticity, and solve it using an own heuristic, which efficiently yields near-optimal results. We demonstrate that correctly considering space elasticity and demand variation is essential. The corresponding impacts on profits and solution structures become even more significant when space elasticity and stochastic demand interact, resulting in up to 5% higher profits and up to 80% differences in solution structures, if both effects are correctly accounted for. We develop an efficient modeling approach, compare the model results with approaches applied in practice and derive rules-of-thumb for planners.

1. Introduction

In line with the objective of maximizing profit or revenue, retailers regularly review their shelf planograms and allocate the assortment to the available shelf space. Retailers have to ensure that each listed product is represented with the right shelf quantity. Shelf-space allocation accordingly determines how many units of each product should be allocated to which shelf [37]. Increasing the shelf quantity of one product results in decreased quantities of other products because shelf space is limited. This impacts visibility and product availability, two of the main drivers behind customer satisfaction [2].

The proliferation of products adds complexity to shelf-space planning, since an increasing number of products are competing for limited shelf space (see e.g., [13,22]). Additionally, most retailers suffer from decreasing space productivity. Gutgeld et al. [28] concluded that 19 of 24 European retailers were unable to maintain their space productivity. A shelf-space optimization model is required to assist retailers with these issues.

Consumers often make decisions at the point of sales, a fact that urges retailers to thoroughly consider relevant demand factors that depend on the respective shelf allocation. Empirical studies show that the number and position of facings (=first unit in the front row of an item on a shelf) are the most important [21,41,14]. First of all, demand for an item increases with a growing number of facings [23]. This phenomenon is called "space-elastic demand".

Furthermore, item demand may also depend on the facing assignments of other items. These cross-product dependencies are referred to as "cross-space elasticity" [15,23]. Finally, the vertical and horizontal positions of items on a shelf also impact customer demand (see e.g., [21]). The more demand is impulse-driven, the more it is space- and position-dependent. Space planning becomes more relevant as the possibility of impacting customer demand by varying the position and number of facings per item increases. This opportunity increases from commodity items, to staples and impulse purchase items (see e.g., [18,19] or [23]). The more demand is impulse-driven, the more it will be subject to variations in demand, and the more important it will be to correctly consider

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stochastic demand in planning. One common characteristic of impulse-driven categories is that consumers choose a favorite item from what they see on the shelf and buy it, if it is better than the no-purchase option. This also means that impulse-driven products are difficult to forecast and usually not substituted [51]. In summary, it can be said that the demand for impulse-driven categories in particular is subject to volatility and impacted by shelf-space decisions, such as the number of facings and the position.

Examples of these impulse-driven categories include in particular seasonal products (e.g., seasonal fruits, Christmas or holiday products, fashion items), promotional products (e.g., "buy one, get one free", product bundles, short-term price offers) or temporary offers (e.g., additional assortments for a limited sales period, pop-up stores, seasonal markets, gondola promotions, promotion area). These categories typically have a limited sales period, cannot be reordered by the retailer during the sales period, perish over time or need to be discounted after the original sales period (see also [3,34]). Because demand for such product types is stochastic, failing to meet it results in either oversupply or shortages. Oversupply leads to higher inventory costs and, in the case of perishable items, to the discounting, salvaging or disposal of left-over items. Shortages lead to lost sales, because the willingness of customers to substitute for impulse-driven products is low or non-existent. This setting is especially applicable to one-time purchases, but not necessarily limited to them [48,33].

Current shelf-space models in the literature center on a deterministic demand model with space-elasticity effects and a restriction in shelf space [37,47]. However, the models have some limitations and one or more of the following drawbacks: (1) the deterministic demand is either expressed in non-linear terms and then solved through heuristics or linearized (either assumed to be linear or piecewisely linearized). This impacts the optimality of solutions and/or the capability to solve problem instances of a size that is relevant in practice. (2) The stochastic nature of the demand is disregarded, and (3) only selected demand effects are accounted for. We therefore develop the first stochastic model that assists retailers in optimizing the number and position of facings, while considering limited shelf space as well as space- and cross-space elasticity and vertical positioning effects. The model balances the trade-off between expected revenues, purchasing, inventory and shortage costs for each product with the aim to maximize profits. The sales period is limited, items cannot be reordered during the sales period and must be disposed of at the end of the period at a salvage value below the initial unit cost, which reduces margins. Typical areas of application are perishable items, such as promotional, seasonal or temporary offers. The model is also applicable to non-perishable items, when no disposal costs are assumed (see also [48]).

The remainder of the paper is structured as follows: Section 2 describes the conceptual background of our research, analyzes the related literature and derives our contribution to research. Section 3 develops the optimization model and shows how it can be solved optimally for cases without and heuristically for cases with cross-space elasticity. Numerical results are presented in Section 4. Section 5 presents our conclusion and an overview of future areas of research.

2. Context of shelf-space planning and the related literature

In this section, we first explain the scope of shelf-space planning and describe associated problems that are inputs or constraints for shelf-space planning (Section 2.1). The decisions to be made in shelf-space planning are defined in Section 2.2 and the associated demand effects analyzed in Section 2.3. This forms the foundation for investigating the related literature and defining the contribution of our work in Section 2.4.

2.1. Scope of shelf space planning and related planning problems

Shelf-space planning and the assignment of items to a shelf are part of the category planning processes. In category planning, Hübner and Kuhn [37], Hübner et al. [38] and Kök et al. [47] differentiate among a series of hierarchical planning steps:

- *Assortment planning* involves (de-) listing products (see e.g., [61,48,68,44,34]) and taking into account substitution effects.
- *Shelf-space planning* assigns the position of items on the shelf and the number of facings to listed products under the constraints of limited shelf size. Actual item demand may depend on the available quantity and position on the shelf (see e.g., [30,64,43]).
- *In-store replenishment planning* determines refill policies. It includes areas such as in-store logistics processes, refill quantities and cycles. Its purpose is to achieve the required on-shelf service levels based on given shelf planograms (see e.g., [69,20,58,4]).

The three planning areas are strongly interdependent, if shelf space and restocking capacity are limited. For example, a broader assortment with more items requires fewer units per item on the shelf or more frequent restocking. Retailers typically solve problems sequentially: first they determine their assortment, next they allocate it to the shelf, and finally they manage in-store replenishment (see [49]). The main reason for this sequential approach is that most retail category planning takes place from a functional perspective. Assortment planning is traditionally the domain of a central marketing planning unit, whereas mid-term shelf-space planning typically is a task of the sales organization. In-store logistics planning is very operational and the store manager is responsible for it. The breakdown into these three planning steps, as applied in practice, helps to overcome the analytical complexities arising from different planning horizons, multiple decision owners and difficulties in demand estimation. Applying such an hierarchical concept that segments the planning steps, supports in developing analytical models that can capture and solve the problem. The planning modules must relate to the organizational hierarchies and responsibilities, as well as to the planning horizon [52,60].

Our research focuses on the shelf-space planning problem. We assume that – according to the aforementioned hierarchical planning concept usually applied by retailers – assortment decisions have been made in the previous step by a different decision owner.

Furthermore, we assume that the retailer applies a direct replenishment policy in each period without backroom storage, which allows for cost-efficient in-store logistics [49,54]. That means that the stores have no, or very limited backroom storage available for these categories. This is because of, for instance, their location in city centers, shopping malls, or because special facilities are required (e.g., refrigeration). The available shelf space for a category is limited and determined exogenously [42,38].

2.2. Decisions to be made in shelf-space planning

A shelf consists of different shelf levels that can be used to position products (see left of Fig. 1). Shelf-space planning allocates items to the different shelf levels and assigns shelf quantities. This includes defining the vertical shelf position (i.e. which shelf level), horizontal position (i.e. which items are next to each other and how far is an item positioned from the aisle) and the number of facings.

Fig. 1 also shows that retailers typically choose a block formation, i.e. they place the same number of facings on several shelf