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Airfoil Design Under Uncertainty Using Non-Intrusive Polynomial Chaos Theory and Utility Functions

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Abstract
Fast and accurate airfoil design under uncertainty using non-intrusive polynomial chaos (NIPC) expansions and utility functions is proposed. The NIPC expansions provide a means to efficiently and accurately compute statistical information for a given set of input variables with associated probability distribution. Utility functions provide a way to rigorously formulate the design problem. In this work, these two methods are integrated for the design of airfoil shapes under uncertainty. The proposed approach is illustrated on a numerical example of lift-constrained airfoil drag minimization in transonic viscous flow using the Mach number as an uncertain variable. The results show that compared with the standard problem formulation the proposed approach yields more robust designs. In other words, the designs obtained by the proposed approach are less sensitive to variations in the uncertain variables than those obtained with the standard problem formulation.

Keywords: Design under uncertainty, stochastic surrogates, utility theory, transonic airfoil design.

1 Introduction

Aerodynamic design plays an important role in the design of various engineering systems where the focus is largely on deterministic approaches (see, e.g., Jameson, 1988; and Ouellet et al., 2004). For example, single-point and multi-point deterministic optimization are widely used. Designs obtained by these approaches often suffer from poor off-design behavior, i.e., their performance degrades in conditions other than the design points. Design under uncertainty (or robust design) aims at designing the system to be insensitive to changes in the design parameters (see, e.g., Zhang et al., 2012; Yao et al., 2011). The formulation of such design problems is, however, not well defined.

This work investigates the use of utility theory (Neumann and Morgenstern, 1947) to rigorously formulate the design under uncertainty problem. Utility theory is a rigorous decision making method

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based on the designer's risk preferences and has not yet been introduced in aerodynamic design. In this paper, utility theory is integrated with efficient methods to compute statistical information to create an efficient and effective aerodynamic design under uncertainty approach. A numerical example of transonic airfoil design is used to illustrate the approach.

2 Problem Formulation

This section describes the standard and the proposed approach to formulating the design problem.

2.1 Standard Formulation

The aim is to design transonic airfoil shapes insensitive to uncertainties in the operational parameters. More specifically, the airfoil performance, such as the drag coefficient, should vary as little as possible for a given range of operational condition. This is often called robust design, or design under uncertainty (see, e.g., Shah et al., 2015). The most widely adopted approach for robust airfoil design is to find the deterministic shape parameters to minimize the sum of the mean drag coefficient and the standard deviation of the drag coefficient subject to constraints on the mean lift coefficient and the airfoil thickness for a range of uncertain operational Mach numbers.

In this work, the problem is formulated using the conventional approach as follows. For the uncertain Mach number M_∞, find the deterministic airfoil shape parameters $\mathbf{x}$ to minimize (Shah et al., 2015)

$$f(\mathbf{x}) = \mu_{Cd}(\mathbf{x}) + \sigma_{Cd}(\mathbf{x}), \tag{1}$$

subject to

$$h(\mathbf{x}) = \mu_{Cl}^{*} - \mu_{Cl}(\mathbf{x}) = 0, \tag{2}$$

and

$$g_j(\mathbf{x}) = t_j^{*} - t_j(\mathbf{x}) \leq 0, \tag{3}$$

where μ_{Cd} is the mean drag coefficient, σ_{Cd} is the standard deviation of the drag coefficient, μ_{Cl} is the mean lift coefficient, μ_{Cl}^{*} the required mean lift coefficient, t_j is the airfoil thickness at location j, with $j = 1$ to m, and t_j^{*} is the minimum airfoil thickness at location j. The airfoil shape parameters have the upper and lower bounds $\mathbf{u}$ and $\mathbf{l}$, respectively, i.e., $\mathbf{x} \in [\mathbf{l},\mathbf{u}]$. It is assumed that the uncertain Mach number is distributed uniformly within the range $M_\infty \in [M_{\infty,l}, M_{\infty,u}]$, where $M_{\infty,u}$, and $M_{\infty,l}$ are the upper and lower bounds on the Mach number, respectively.

2.2 Formulation by Utility Functions

An alternative formulation to the conventional approach of (1)-(3) is investigated. In particular, airfoil design under uncertainty is formulated based on utility theory (Neumann and Morgenstern, 1947). Utility theory translates a range of targeted responses, due to uncertainty in operational parameters, at a specific design point, using a specific function, called the utility function, to metrics used for design comparison. Within the utility function, one can change a parameter to obtain a given risk preference of the designer-called risk aversion (or risk avoiding), risk neutral, and risk loving (risk taking) of the targeted function. These three types of risk preferences are shown graphically in Fig. 1.