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Robust Self-Tuning Control under Probabilistic Uncertainty using Generalized Polynomial Chaos Models

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Abstract: A robust self-tuning controller for a chemical process is developed based on a generalized Polynomial Chaos (gPC) model that accounts for probabilistic time-invariant uncertainty. Using this model, it is possible to calculate analytical expressions of the one-step ahead predicted mean and variances of controlled and manipulated variables. The key idea is to consider these predicted values for performing online robust tuning of the controller through a quadratic optimization procedure. The gPC model is also used to identify overlap between consecutive probability density functions (PDFs) of manipulated variables and to find trade-offs between the aggressiveness of the self-tuning controller and robustness to uncertainty based on this overlap. The proposed methodology is illustrated by a continuous stirred tank reactor (CSTR) system with stochastic variations in the inlet concentration. The efficiency of the proposed algorithm is quantified in terms of control performance and robustness.

Keywords: Self-tuning controller, uncertainty propagation, quadratic optimization

1. INTRODUCTION

The Proportional-Integral-Derivative (PID) controller is one of the most commonly used controller in the industry due to its simple structure. When applied to linear time invariant processes, optimal tuning rules for PIDs are available. However, chemical processes generally exhibit nonlinear or time-varying behaviour thus requiring a different approach for tuning to achieve satisfactory closed loop performance. Self-tuning controllers have been often suggested as a suitable alternative to deal with nonlinear and time-varying behaviours. Although this work deals with self-tuning PID or PI controllers the techniques can be easily extended to other control laws.

The basic idea of self-tuning controllers is to recursively identify the best model of the process from closed loop input-output data and to subsequently tune the controller based on the identified model. Much effort has been devoted to the development of self-tuning controllers. Early work includes pole placement (Astrom & Wittenmark, 1980) and minimum variance control (Agarwal & Seborg, 1987) approaches. Recently, a constrained optimization was formulated to find the tuning PID parameters (Leva & Maggio, 2011). The trade-off between control performance and robustness has been studied by graphic means for both PI and PID controllers (Garpinger, et al., 2014). However, these reported approaches do not account for model uncertainty that will result in deterioration of closed loop performance.

Model uncertainty may originate from either intrinsic time varying phenomena, e.g. unmeasured disturbances, model structure error or may result from innacurate model identification from noisy data. Little work has been done to develop robust self-tuning PID algorithms in the presence of uncertainty. Using empirical Gaussian Process models, self-tuning algorithms were developed (Sbarbaro & Murray-Smith, 2005, Chan, et al., 2016). These empirical model-based algorithms are sensitive to the amount and density of available data and the validity and complexity of selected models. In contrast, first-principle models have generally better extrapolation properties and their accuracy is often less sensitive to the amount and quality of available data, as compared to empirical models. However, the performance of a controller that is based on a first-principle model can be significantly degraded due to uncertainty. Normally, robust controllers are designed with respect to worst case uncertainty thus resulting in conservatism. This can unnecessarily degrade performance, since worst case scenarios have low probability of occurrence (Zhang, et al., 2016).

The current paper addresses the limitations outlined above by proposing an online robust self-tuning control algorithm based on a first-principle model with model uncertainty. The key idea is to approximate the probability density function (PDF) describing the uncertainty and propagate uncertainty onto one-step ahead predictions of the means and variances of the controlled and manipulated variables.

Since the self-tuning algorithm has to be executed online, it is very important to propagate the uncertainty onto the manipulated and controlled variables in a computationally efficient manner. Although such calculations could be done with a Monte Carlo sampling based approach, this is prohibitive for online implementation of a self-tuning

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controller. Instead, the generalized polynomial chaos (gPC) expansion (Xiu, 2009) is used in this work. The gPC has been studied by a few authors in different modeling (Nagy & Braatz, 2007), optimization (Mandur & Budman, 2014) and fault detection problems (Du, et al., 2014; Du, et al., 2016; Du, et al., 2015). The advantage of gPC is that it can efficiently propagate non-standard probabilistic distributions of model uncertainty onto the predictions of manipulated and controlled variables. Using gPC, statistical moments such as variances of the model outputs can be quickly computed. Then, using the one-step ahead predictions of the means and variances of controlled and manipulated variables, the optimal on-line tuning of the PID controller is formulated as a quadratic optimization problem, subject to constraints on the rate of change of the controller tuning parameters. The latter constraint is based on the overlap between two consecutive PDFs of the controlled variable.

This paper is organized as follows. Section 2 presents the background and the principal methodology used in this paper. The formulation of a quadratic optimization for robust tuning of the controller is presented in Section 3. The method is illustrated for an endothermic continuous stirred tank reactor (CSTR) in Section 4. Analysis and discussion of the results are given in Section 5 followed by conclusions in Section 6.

2. UNCERTAINTY PROPAGATION WITH GENERALIZED POLYNOMINAL CHAOS

A gPC expansion represents a random variable of arbitrary PDF as a function of another random variable ξ of a known prior distribution (Xiu, 2009). To preserve orthogonality, the basis functions in the gPC expansion are selected according to the choice of the distribution of ξ. Assuming a nonlinear system described as follows:

$$\dot{\mathbf{q}} = h(t, \mathbf{q}, u; \mathbf{g}) \tag{1}$$

, where $0 \le t \le t_f$, $\mathbf{q} \in R^n$ contains the system states with initial conditions $\mathbf{q}(0) = \mathbf{q}_0$ over time domain $[0, t_f]$, $\mathbf{g} \in R^{ng}$ defines an unknown time varying input vector, u is the manipulated variable. The PID control algorithm is defined as follows:

$$u = u_s + K_p e + (K_p/\tau_i)\int_0^t e\, dt' + K_p \tau_d \frac{de}{dt} \tag{2}$$

, where e is the error ($e=q_{c,set}-q_c$), K_p, τ_i and τ_d are the controller parameters. The state q_c is the controlled variable, which for simplicity is assumed to be one of the elements of $\mathbf{q}$. The states in this current study are assumed to be measurable, otherwise an observer is required. The model uncertainty is captured through the stochastic input $\mathbf{g}$ using a gPC approximation. Each unknown parameter g_i ($i=1,2,\ldots,n_g$) in $\mathbf{g}$ is assumed to be a function of a set of random variables $\boldsymbol{\xi} = \{\xi_i\}$:

$$g_i = g_i(\xi_i) \tag{3}$$

,where ξ_i is the i^{th} random variable. The random variables $\{\xi_i\}$ are assumed to be independent and of equal distributions. It should be noted that although ξ_i is assumed to follow a standard distribution (Xiu, 2009); elements $\{g_i\}$ of $\mathbf{g}$ may follow any non-standard distribution. An augmented vector of states is defined to include both the system states $\mathbf{q}$ and the manipulated variables u as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{u} \end{bmatrix} = f(t, \mathbf{x}; \mathbf{g}) \tag{4}$$

The unknown $\mathbf{g}$ and states $\mathbf{x}=\{x_1,\ldots, x_{n+1}\}$ are approximated in terms of polynomial orthogonal basis functions $\Phi_k(\boldsymbol{\xi})$ as:

$$g_i(\xi) = \textstyle\sum_{k=0}^{\infty} g_{i,k}\Phi_k(\xi) \tag{5.a}$$

$$x_j(\xi) = \textstyle\sum_{k=0}^{\infty} x_{j,k}(t)\Phi_k(\xi) \tag{5.b}$$

, where $\{x_{j,k}\}$ are the gPC coefficients of the j^{th} states and the manipulated variable u at each time instant t, and $\{\Phi_k(\xi)\}$ are multi-dimensional orthogonal polynomial basis functions. For practical application, (5.a) and (5.b) are truncated into a finite number of terms. It is assumed that there is prior information about the disturbances and thus the gPC coefficients in (5.a) can be estimated. However, the gPC coefficients of $\{x_{j,k}\}$ in (5.b) are unknown. This can be calculated by substituting (5.a) into the model of (4), i.e., the combination of (1) and (2), by using Galerkin projections with respect to each polynomial chaos function $\{\Phi_k(\xi)\}$ as (Xiu, 2009):

$$\left\langle \dot{x}_j(t,\xi), \Phi_k(\xi) \right\rangle = \left\langle f(t, x_j(t,\xi), u(t), g(\xi)), \Phi_k(\xi) \right\rangle \tag{6}$$

The total number of terms (P) in (6) is a function of the number of terms p in (5.a) that is necessary to represent the a priori known distribution of $\{g_i\}$ and the number of elements n_g in $\mathbf{g}$.

$$P = ((n_g + p)!/(n_g!\,p!)) - 1 \tag{7}$$

The inner product in (6) between two vectors $\varphi(\xi)$ and $\varphi'(\xi)$ is defined as:

$$\left\langle \varphi(\xi), \varphi'(\xi) \right\rangle = \int \varphi(\xi)\varphi'(\xi) w(\xi) d\xi \tag{8}$$

, where the integration is conducted over the entire domain of the random variables ξ, and $w(\xi)$is a weighting function, which is chosen for normalization purposes with respect to the type of polynomial basis functions used in the expansion (Xiu, 2009). The statistical moments of $\mathbf{x}$ can be calculated at any given time instant t as a function of the gPC coefficients $\{x_{j,k}\}$ in (5.b) with the following analytical formulae:

$$\mathrm{E}\left(x_j(t)\right) = \mathrm{E}\left[\textstyle\sum_{k=0}^{P} x_{j,k}(t)\Phi_k\right] = x_{j,0}(t)\mathrm{E}[\Phi_0] + \textstyle\sum_{k=1}^{P}\mathrm{E}[\Phi_k] = x_0(t) \tag{9}$$

$$\mathrm{var}\left(x_j(t)\right) = \mathrm{E}\left[\left(x_j(t) - \mathrm{E}\left(x_j(t)\right)\right)^2\right] = \mathrm{E}\left[\left(\textstyle\sum_{k=0}^{P} x_{j,k}(t)\Phi_k - x_0(t)\right)^2\right]$$

$$= \mathrm{E}\left[\left(\textstyle\sum_{k=1}^{P} x_k(t)\Phi_k\right)^2\right] = \textstyle\sum_{k=1}^{P} x_k(t)^2\mathrm{E}\left(\Phi_k^2\right) \tag{10}$$

The availability of analytical expressions for the mean and variance as per equation (9) and (10) is the main rationale for using the gPC, since these quantities have to be repeatedly calculated online during the closed loop operation of the self-tuning controller.

3. SELF-TUNING CONTROLLER BASED ON INTERNAL GPC MODEL

The parameters of the PID controller are adjusted by a tuning algorithm based on a gPC model and a quadratic optimization (QP) described below.