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Oscillatory Global Output Synchronization of Nonidentical Nonlinear Systems ⋆

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Abstract: In this work, we study a global output synchronization problem for nonidentical nonlinear systems having relative degree 2 or higher. The synchronization uses a partial projection of individual subsystems into the Brockett oscillators. Our approach is based on output feedback and uses a higher order sliding mode observer to estimate the states and perturbations of the synchronized nonlinear systems. Simulation results are provided to illustrate the performance of the proposed synchronization scheme.

Keywords: Global synchronization; nonidentical nodes; Brockett oscillator; nonlinear systems; higher order sliding modes.

1. INTRODUCTION

Over the last decade, the synchronization of complex dynamical systems and/or network of systems has attracted a great deal of attention from multidisciplinary research communities due to their pervasive presence in nature, technology and human society [Blekhman (1988); Pikovsky et al. (2003); Strogatz (2003); Osipov et al. (2007)]. Among potential application domains of synchronization, it is worth to mention the smooth operations of microgrid [Efimov et al. (2016); Schiffer et al. (2014)], secure communication [Fradkov et al. (2000); Fradkov and Markov (1997)], formation control [Andrievsky and Tomashevich (2015)], chaos synchronization [Rodriguez et al. (2008, 2009)], genetic oscillators [Efimov (2015); Ahmed et al. (2015)], *etc.*

Significant progress has been made during the past decade in the area of control design for synchronization, consensus or motion coordination, the existing literature is huge and covers wide area of topics [Gazi and Passino (2011); Shamma (2007); Olfati-Saber and Murray (2004); Olfati-Saber et al. (2007); Panteley and Loria (2017)]. Until now, a large number of works are available on the problem of synchronization of networks with identical nodes, particularly when the nodes are linear time-invariant systems [Scardovi and Sepulchre (2009); Olfati-Saber et al. (2007); Tomashevich (2017)]. However, most physical systems are often not identical and frequently they are nonlinear in nature. The behavior of dynamical networks with nonidentical nodes is much more complicated than the identical-node case. Usually, no common equilibrium for all nodes exists even if each isolated system has an equilibrium, the same for other invariant solutions which can be destroyed or created by synchronization protocols.

The study of synchronization of dynamical networks with nonidentical nodes is complicated and very few results have been reported by now [Hill and Zhao (2008)]. In [Sun and Geng (2016)], adaptive synchronization has been proposed for nonidentical linear systems. Several collective properties for coupled nonidentical chaotic systems were respectively discussed in [Osipov et al. (1997); Fradkov and Markov (1997); Rodriguez et al. (2009, 2008); Plotnikov et al. (2016)]. Synchronization for smooth and piecewise smooth nonidentical systems with application to Kuramoto oscillators has been studied in [DeLellis et al. (2015)]. Controlled synchronization for two coupled hybrid FitzHugh-Nagumo systems has been studied in [Plotnikov and Fradkov (2016)]. In [Ahmed et al. (2016d)], the authors have proposed robust synchronization for identical/nonidentical multi-stable systems. However, the systems are assumed to admit a decomposition without cycles (neither homoclinic nor heteroclinic orbits). Recently, the results of [Ahmed et al. (2016d)] have been applied to a multi-stable oscillator model [Ahmed et al. (2016b,c, 2017)].

The goal of this work is to address the issue of synchronization of nonidentical nonlinear systems using output feedback only, and an additional subgoal is to have an oscillatory behavior in the synchronized state. Since many engineering systems have relative degree 2 and higher (*e.g.*, pendulum systems [Davila et al. (2005)], oscillators [Rodriguez et al. (2009)], robot manipulators [Salgado et al. (2014)], DC motor [Khalil (2014)]), the particular focus is put on this class of systems. Studying nonidentical systems in general setting, we will assume that neither an equilibrium for each isolated node nor a synchronization manifold exists, so to synchronize them it is necessary

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to apply a feedback transformation [Khalil (2014)] that projects all subsystems to a common dynamics that can be synchronized next. In this work, the Brockett oscillator model is selected for this purpose. This is motivated by a global synchronization control recently proposed for such systems in [Ahmed et al. (2016c)]. Then higher order sliding mode observer is applied to estimate the unmeasurable states and perturbations using the idea presented in [Fridman et al. (2008)]. In short, the main idea is to compensate the nonlinearities of individual systems followed by a nonlinear injection converting some parts of the systems into the Brockett oscillator form. The only restriction is that the individual systems should have relative degree 2 or higher (see Appendix for the definition), but most of the popular oscillator models satisfy this criteria if the output signal is properly selected.

The rest of the article is organized as follows: Section 2 gives the problem statement followed by the synchronizing control design in Section 3. In Section 4, numerical simulation results are given and finally Section 5 concludes this article. Preliminaries on relative degree and a summary of the result of [Ahmed et al. (2016c)] can be found in the Appendix.

2. PROBLEM STATEMENT

The following family of affine controls nonlinear systems is considered in this work for $i = \overline{1, N}$ with $N > 1$:

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i,$$
$$y_i = h_i(x_i), \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state, $u_i \in \mathbb{R}$ ($u_i : \mathbb{R}_+ \to \mathbb{R}$ is locally essentially bounded and measurable signal) is the input, $y_i \in \mathbb{R}$ is the output; $f_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$, $h_i : \mathbb{R}^{n_i} \to \mathbb{R}$ and $g_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$ are sufficiently smooth functions. Denote the augmented state vector of (1) as $x = [x_1^T, \ldots, x_N^T]^T \in \mathbb{R}^n$ with $n = \sum_{i=1}^{N} n_i$, $y = [y_1, \ldots, y_N]^T \in \mathbb{R}^N$ as the augmented output, and $u = [u_1, \ldots, u_N]^T \in \mathbb{R}^N$ as the augmented input. The relative degree condition (see Appendix) imposed on system (1) is summarized by the following assumptions:

Assumption 1. For all $i = \overline{1, N}$, the systems in (1) have global relative degree $r_i \in [2, n_i]$ and globally defined normal form [Marino and Tomei (1996)].

Under this assumption, for each subsystem in (1) there is a diffeomorphic transformation of coordinates $T_i : \mathbb{R}^n \to \mathbb{R}^n$ such that [Marino and Tomei (1996); Khalil (2014)]:

$$\begin{bmatrix} \eta_i \\ \xi_i \end{bmatrix} = T_i(x_i),$$

where $\xi_i \in \mathbb{R}^{r_i}$ and $\eta_i \in \mathbb{R}^{n_i - r_i}$ are new components of the state, and for all $i = \overline{1, N}$ the i^{th} subsystem of (1) can be represented in the normal form:

$$\dot{\eta}_i = \varphi_i(\eta_i, \xi_i), \tag{2}$$
$$\dot{\xi}_i = A_{r_i}\xi_i + b_{r_i}[\alpha_i(\xi_i) + \beta_i(\xi_i)u_i], \tag{3}$$
$$y_i = c_{r_i}\xi_i,$$

where $\varphi_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$, $\alpha_i : \mathbb{R}^{n_i} \to \mathbb{R}$ and $\beta_i : \mathbb{R}^{n_i} \to \mathbb{R}$ are smooth functions, β_i is separated from zero, and

$$A_{r_i} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ 0 & 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}, \; b_{r_i} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$
$$c_{r_i} = [\, 1 \; 0 \ldots 0 \,]$$

are in the canonical form. The subsystem (2) is called the *zero dynamics* of i^{th} subsystem in (1), which we assume to be robustly stable:

Assumption 2. For all $i = \overline{1, N}$, the systems in (2) are input-to-state stable (ISS) with respect to the inputs ξ_i [Angeli and Efimov (2015); Dashkovskiy et al. (2011)].

Concerning the definitions of ISS property used in this work, we will not distinguish ISS with respect to a set in the conventional sense [Dashkovskiy et al. (2011)] or for a multistable system [Angeli and Efimov (2015)], the only property we need here is boundedness of the variables η_i for bounded ξ_i. More detailed analysis of the possible asymptotic behavior in (2) for the latter scenario is presented in [Forni and Angeli (2015)].

Then the synchronization problem consists in finding a control u such that the members of the family (1) follow each other. Since the states of the subsystems in (1) may have different dimensions n_i, a state synchronization error $x_i - x_j$ cannot be defined in general (*i.e.* the states of the subsystems in (1) cannot follow their neighbors), but an output synchronization can be formulated:

Definition 1. The family (1) exhibits a *global output synchronization* if

$$\lim_{t\to\infty} (y_i(t) - y_j(t)) = 0, \quad \forall i, j = \overline{1, N}$$

for any initial conditions $x_i(0) \in \mathbb{R}^{n_i}$, $i = \overline{1, N}$.

Note that under Assumption 1 an additional requirement can be imposed on synchronization of derivatives:

$$\lim_{t\to\infty} \{\dot{y}_i(t) - \dot{y}_j(t)\} = 0 \quad \forall i, j = \overline{1, N}.$$

An output feedback controller has to be designed to achieve the global output synchronization for (1).

3. SYNCHRONIZATION CONTROL DESIGN

The idea of this work is to design a feedback controller that will convert a part of subsystems (3) into the form of the Brockett oscillator [Brockett (2013)] through nonlinearity injection. Then global synchronization results can be easily obtained using the control proposed in [Ahmed et al. (2016c)] (a summary is given in Appendix). However, this controller requires all components of the state vector to be available, which limits its implementation. Therefore, to overcome this difficulty, a high-order sliding-mode observer is used.

To simplify the presentation of the forthcoming synchronization protocol design, let us assume that

$$u_i + d_i = \alpha_i(\xi_i) + \beta_i(\xi_i)u_i,$$

where $d_i \in \mathbb{R}$ is a new disturbance signal in (3) for each $i = \overline{1, N}$ (since β_i is not singular, such a representation always exists).

Assumption 3. For all $i = \overline{1, N}$, the unknown input $d_i : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable for almost all $t \geq 0$, and there is a constant $0 < \nu^+ < +\infty$ such that $\operatorname{ess\,sup}_{t\geq 0} |\dot{d}_i(t)| \leq \nu^+$.