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IFAC PapersOnLine 50-1 (2017) 2523–2529

Further Improvement of Fixed-Time Protocol for Average Consensus of Multi-Agent Systems ⋆

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Abstract: In this paper, a new class of fixed-time consensus protocols are proposed to further improve the existing fixed-time consensus protocols. Different from the existing fixed-time consensus protocols, both design parameters and the selected protocol function of the presented protocols determine the convergence rate and control input. Therefore, the presented protocol provides more flexibility to improve convergence rate and reduce control input. It is proved that the presented protocol can achieve fixed-time average consensus for multi-agent system with interaction topology modeled as undirected and directed graph. Numerical simulations are performed to demonstrate that the presented protocol can improve the convergence rate of the existing fixed-time consensus protocols without increasing control input.

Keywords: Fixed-time consensus protocol, multi-agent system, average consensus, directed graph, undirected graph, convergence rate, control input

1. INTRODUCTION

Recently, consensus problem of multi-agent systems has received great attention and considerable efforts from researchers in many fields. The consensus of multi-agent systems means that states of all the agents reach agreement on a common desired quality by implementing an appropriate consensus protocol, which only depends on its own information and its neighbor's information.

In the study of consensus problem, convergence speed is an important performance index. It has been shown that algebraic connectivity (or the second smallest eigenvalue) determines the convergence rate of consensus for multi-agent system (see Olfati-Saber and Murray (2004)). Specifically, faster convergence rate requires larger algebraic connectivity. This finding motivates some researches to improve the consensus by increasing the algebraic connectivity (see Kim and Mesbahi (2006), Olfati-Saber (2005)). Although these results improve convergence rate via linear protocol, the consensus can never be reached within finite time. Compared with asymptotical consensus, finite time consensus can achieve exact consensus and has better disturbance rejection property and stronger robustness against uncertainties. Therefore, finite time consensus problem has been promoted to achieve high speed convergence. Cortes (2006) proposed the normalized and signed gradient descent flow of a differentiable function to solve finite time network consensus problem. With the help of homogeneous method, Wang and Hong (2008) presented finite time consensus protocol for second order multi-agent system. Based on adding a power integrator method, Li et al. (2011) constructed finite time consensus algorithm for leaderless and leader-follower multi-agent system with external disturbance. Using terminal sliding mode technique, Khoo et al. (2009) investigated finite time consensus problem of multi-robot system. It is worth noting that the convergence time of finite time consensus protocol depends on initial condition. However, in many practical applications, it is always hard to obtain accurate information of initial condition, which may bring difficulties in giving an accurate estimate of settling time. In addition, the convergence time of finite time consensus grows unboundedly with the increase of the initial condition.

Fixed-time stability proposed by Polyakov (2012) is an extension of finite time stability and it can guarantee convergence within finite time upper bounded by a constant independent of initial condition. Due to this attractive feature, fixed-time stability has been applied to design uniform robust exact differentiator (see Cruz-Zavala et al. (2011)), design power system stable controller (see, Ni et al. (2016), Ni et al. (2017), for instance) and address consensus problem for multi-agent system (see, e.g., Zuo and Tie (2014), Zuo (2015), Zuo et al. (2014), Defoort et al. (2015), Zuo and Tie (2016), Fu and Wang (2016), Meng and Zuo (2016)). Fixed-time consensus problem has been solved for multi-agent systems in a network with undirected topology (see Zuo and Tie (2014)), directed topology (see Zuo (2015)), and directed and switching topology (see Zuo et al. (2014)). In Defoort et al. (2015) and Zuo and Tie (2016) , fixed-time robust consensus problem was addressed for multi-agent system with unknown dynamics and external disturbances. In Fu and Wang (2016), fixed-time coordinate tracking problem with relative output and state measurement was studied for

⋆ This work was supported in part by the National Natural Science Foundation of China under Grants 51177117, 51307130, by the Creative Research Groups Fund of the National Natural Science Foundation of China under Grant 51221005.

10.1016/j.ifacol.2017.08.064

multi-agent system with input uncertainties. In Meng and Zuo (2016), fixed-time consensus was extended to multi-agent system with mixed cooperative and antagonistic interactions. Nevertheless, the amplitude of control input is required to be rather high to achieve fixed-time consensus for those agents whose initial states are far away from consensus states. This is a natural consequence of fixed-time property, as pointed out by Polyakov et al. (2015). However, the convergence rate and control input of the existing fixed-time consensus protocols are only determined by design parameters, which means that faster convergence can only be achieved at the cost of higher control input, which prohibits its further development. How to find a protocol to improve the convergence rate without increasing the control input remains unsolved for fixed-time consensus.

In this paper, we attempt to solve the problem and improve the existing fixed-time consensus protocols by presenting a new class of fixed-time consensus protocols. It is worth noting that besides design parameters, the selected protocol function can also influence the convergence time and the amplitude of control input, which provides more flexibility to improve convergence performance and reduce control input. It is proved that the presented protocol can achieve fixed-time average consensus for multi-agent system in a network with undirected and directed topology. The presented protocol can improve the convergence rate of the existing fixed-time consensus protocols without increasing the control input, which is verified through numerical simulations.

The rest of this paper is organized as follows. Section 2 formulates the problem and reviews preliminary knowledge necessary throughout the paper. Main results of this paper are presented in Section 3 and simulation results verifying the effectiveness of proposed controller are given in Section 4. Finally, the conclusion is drawn in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Graph Theory

Let $G(A) = \{V, \varepsilon, A\}$ be a graph with a node set $V(G) = \{v_1, v_2, ..., v_N\}$, an edge set $\varepsilon(G) \subseteq V \times V$ and an adjacency matrix $A = [a_{ij}] \in R^{N\times N}$. For directed graph, an edge (v_i, v_j) denotes directed communication link from node v_i to node v_j, but not necessarily vice versa. In contrast, for undirected graph,$(v_i, v_j) \in \varepsilon$ implies $(v_j, v_i) \in \varepsilon$, i.e., the nodes v_i and v_j can send information to each other. If $(v_i, v_j) \in \varepsilon$, node v_i is called the neighbor node of node v_j or v_i and v_j are said to be adjacent. The index set $N_j = \{i : (v_i, v_j) \in \varepsilon\}$ includes the nodes index of all neighbors of v_j. For undirected graph, $A = [a_{ij}]$ is the adjacency matrix with nonnegative elements $a_{ij} = a_{ji} \geq 0$. Moreover, $a_{ij} > 0$ if $(v_i, v_j) \in \varepsilon$, $a_{ij} = 0$ if $(v_j, v_i) \notin \varepsilon$, and $a_{ii} = 0$ for $i \in I_N = \{1, ..., N\}$. For directed graph, the definition of the adjacency matrix $A = [a_{ij}]$ is similar except $a_{ij} = a_{ji}$. A sequence of distinct vertices $\{v_i, ..., v_j\}$ is called a path from node v_i to node v_j if the consecutive vertices of the sequence are adjacent. A directed graph is called strongly connected if and only if for any nodes v_i,v_j, there exists a directed path from i to j . An undirected graph is called connected if there exists a path for any two nodes.

Definition 1:(Olfati-Saber and Murray (2004)) Consider a weighted directed graph $G(A) = \{V, \varepsilon, A\}$. Let $\widetilde{\varepsilon}$ be the set of reverse edges of $G(A)$ obtained by reversing the order of nodes of all pairs in ε. $G(\hat{A}) = \{V, \hat{\varepsilon}, \hat{A}\}$ denotes the mirror of $G(A)$ with the same node set as $G(A)$, the set of edges $\hat{\varepsilon} = \varepsilon \cup \widetilde{\varepsilon}$, and the symmetric adjacency matrix $\hat{A} = [\hat{a}_{ij}]$ with nonnegative elements $\hat{a}_{ij} = \hat{a}_{ji} = (\omega_i a_{ij} + \omega_j a_{ji})/2 \geq 0$.

2.2 Fixed-time Stability

Consider the following differential equation system:

$$\dot{x}(t) = f(x(t)), \quad x(0) = x_0. \tag{1}$$

where $x \in R$ and $f : R_+ \times R^n \rightarrow R^n$ is a nonlinear function. Suppose that the origin is an equilibrium point of (1).

Definition 2:(Defoort et al. (2015), Bhat and Bernstein (2000)) The origin of system (1) is a finite time stable equilibrium if the origin is Lyapunov stable and there exists a function $T : R^n \rightarrow R^+$, called the settling time function, such that for every $x_0 \in R^n$, the solution $x(t, x_0)$ of system (1) is defined on $[0, T(x_0))$, $x(t, x_0) \in R^n$, for all $t \in [0, T(x_0))$, and $\lim\limits_{t\rightarrow T(x_0)} x(t, x_0) = 0$.

Definition 3: (Polyakov (2012)) The origin of system (1) is said to be a fixed-time stable equilibrium point if it is globally finite-time stable with bounded convergence time $T(x_0)$, that is, there exists a bounded positive constant $T_{\max}$ such that $T(x_0) < T_{\max}$ satisfies.

Lemma 1. (Zuo and Tie (2014)) Consider the following differential equation:

$$\dot{y} = -\alpha y^{2-\frac{p}{q}} - \beta y^{\frac{p}{q}}, y(0) = y_0 \tag{2}$$

where α,$\beta > 0$, p,q are positive odd integers satisfying $p < q$. Then, the equilibrium point of system (2) is fixed-time stable and the settling time is upper bounded by:

$$T \leq \frac{q\pi}{2\sqrt{\alpha\beta}(q-p)} \tag{3}$$

2.3 Problem Formulation

Consider a group of N first order agents with the following dynamics:

$$\dot{x}_i = u_i, \quad i \in I_N = \{1, ..., N\} \tag{4}$$

where x_i is the state of agent i and u_i is the protocol to be designed. With a given protocol u_i , the multi-agent system is said to achieve consensus if for any i,$j \in I_N$, $|x_i(t) - x_j(t)| \rightarrow 0$ as $t \rightarrow \infty$. The multi-agent system is said to achieve fixed time consensus if for any $i, j \in I_N$ and initial condition $x_i(0)$, there exists a constant T_{max} independent of initial condition $x_i(0)$ such that the settling time $T \leq T_{max}$ and $\lim_{t\rightarrow T} |x_i(t) - x_j(t)| \rightarrow 0$, $x_i(t) = x_j(t), for\ t \geq T$. In addition, if the final agreement value satisfies $\lim_{t\rightarrow T} x_i(t) = \frac{1}{N}\sum_{j=1}^{N} x_j(0)$, then the multi-agent system achieves fixed-time average