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Redistributing stock in library systems with a depot

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ABSTRACT

Public library organizations often utilize depots for carrying out shipments to libraries in case of stock-outs and for storing low demand rental items at low cost. Similar systems may be employed by rental companies for other rental products such as tools, DVDs, and jewelry. Since shipments deplete the depot's inventory, stock must be taken back from the libraries in order to deal with future shipment requests. These shipment and take-back operations are carried out periodically, e.g. daily or weekly. This work focuses on optimizing the decisions for shipments and take-backs. We model the system by means of a Markov decision process and investigate its optimal policy for various problem instances. For the take-back decision, we distinguish between so-called threshold, reactive, and preventive take-backs. We use the insights from the MDP to develop a three-phase take-back heuristic. In experiments, our heuristic performs within 1% on average from the optimal solution. For settings with a large number of libraries, it is shown that an acceptable performance can be achieved by setting a base-stock level at the depot and taking back sufficient stock from the libraries to achieve this level.

1. Introduction

In recent years, the Dutch public library system is increasingly adopting concepts derived from e-commerce. As of 2016, clients can access and order items online from a nationwide catalog showing all the items and their availability at each library in the country. The fulfillment of these online orders is a challenging task, in particular when clients request items for pickup from a library that does not have the item in stock. In order to satisfy such requests, libraries often introduce a joint depot dedicated to shipment of locally unavailable items.

Currently, public libraries in many countries ship directly between libraries in response to stock-outs. However, the number of items in a shipment is typically small, leading to ineffective usage of transportation devices. This fact has become increasingly pressing, since demand for physical books has significantly decreased over time after the introduction of internet and e-books. A depot allows to consolidate these small shipment streams into larger streams. This is easier to coordinate and creates significant economies of scale, since items can be shipped using fewer transportation devices and handled in one dedicated place. In addition, a depot can serve as a low cost storage location for items that are currently not needed. For this reason library organizations are increasingly adopting a system with shipments from a depot.

The above motivation applies not only to public libraries but also to rental systems in general. For example, tool rental companies store rare and expensive tools in a depot so that they can be shared effectively between rental locations. Other possible rental products for which a depot may be utilized are jewelry and DVDs. While the focus application of this article is public libraries, insights and heuristics carry over analogously to those other rental systems.

In library systems with a depot, various operational decisions are carried out periodically. When locally unavailable items are requested at a library, these are shipped from the depot. In contrast to sales-driven companies, where stock is bought and sold, stock in library systems is often fixed and all rented items are returned by the client. The depot will thus have to be resupplied by carrying out a take-back operation of items from the individual libraries. The main difficulty lies in deciding how many items to take back in total and from which libraries. Since due to budget cuts the government funding for public libraries has significantly decreased in the last several years, it is important to carry out these operations efficiently.

An often encountered practical problem for public libraries is that a large part of the collection consists of low-demand items. Muckstadt and Thomas (1980) conclude that two-echelon systems are important for low-demand items. Hence, storage of such items in a low-cost depot may be an effective strategy to reduce holding costs and free up space at the libraries for other items or activi-

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ties. An important problem is deciding which low-demand items to store in the depot.

In this paper we simultaneously optimize the decisions for resupplying the depot and dealing with low-demand items at the libraries. We consider a periodic review model where demands and returns at the libraries occur between reviews. At the review, stock is observed and there is an option to carry out shipments and take-backs. By first solving an MDP for a problem with a single library and single depot, we obtain the main insights for storing low-demand items at the depot. Subsequently, we solve MDPs for problems with multiple libraries. By analyzing the optimal policy for several example configurations, we obtain insights into optimal shipment and take-back operations. Based on the insights we formulate a near-optimal heuristic for larger problem instances and in various experiments we compare it to the optimal policy and several other simple heuristics.

The research on multilocation rental systems has mainly focused on vehicle rentals systems (Ernst et al., 2011; Li and Tao, 2010). For vehicle rentals, the common option for dealing with stock-outs is to provide substitute vehicles, whereas for library books shipments can be a practical option because the items are easily shipped and clients are typically willing to wait for a shipment. In vehicle rentals, shipments from another location in response to demand are typically only considered in deterministic problems. For example, Ernst et al. (2011) determine an optimal schedule for a finite planning horizon where bookings are known in advance. For a bike sharing system, Dell'Amico et al. (2016) solve a single period rebalancing problem. Such deterministic methods do not match the multiperiod stochastic setting that we consider.

In stochastic settings, the main focus in vehicle rental literature is on optimizing the fleet size and the fleet redistribution policy, often by applying queuing theory (George and Xia, 2011). Li and Tao (2010) use dynamic programming to optimize the redistribution policy. The authors consider a two-location system where vehicles rented from one location can be returned to the other location in the same period. While we do not consider returns to other locations, we add a depot, more than two locations, and consider stochastic rental times. Our MDP therefore has a significantly larger state space than the dynamic program in Li and Tao (2010) and requires an efficient implementation. For a multilocation library system with lateral transshipments, Van der Heide and Roodbergen (2013) apply dynamic programming to optimize lateral shipments and stock redistribution policies. They show that a dynamic redistribution policy, accounting for current on-hand and rented stock at each library, significantly outperforms the standard policy in practice of sending back each item to its owner location. To the best of our knowledge, no authors have considered the redistribution of stock in a library system with a depot.

Shipments from a depot in response to stock-outs have been studied recently in spare parts inventory control. In a case study for the spare parts division of a car manufacturer, Axsäter et al. (2013) demonstrate that significant cost savings can be achieved by introducing the shipment option. Van Wijk et al. (2013) derive structure results for the optimal operational decisions of assigning shipment requests from the depot to local stock points. In the above papers, stock is transferred from the depot to local stock points, but no attention is paid to the transfer of returned stock from local stock points to the depot, which is an important feature of library systems.

Hub-and-spoke systems are also characterized by exchanges of vehicles between locations and a depot. In a hub-and-spoke system, vehicles rented at the hub return at the spoke, and vice versa, while in the library system items are typically rented from and returned to the same location. These essentially different dynamics demand different strategies for repositioning stock. Köchel (2007) and Song and Carter (2008) consider repositioning of empty cars in hub-and-spoke systems. Both authors start by considering repositioning policies for systems with a hub and a single spoke. The resulting policies are used to formulate heuristics for systems with multiple spokes. We follow a similar approach by basing part of our heuristic on the optimal policy of the single library problem.

The outline of the article is as follows. Section 2 introduces the model for the library system with a depot. In Section 3 MDPs are solved for base scenarios with one, two, and three libraries to gain insight into the relevant trade-offs. In Sections 4 and 5 shipment and take-back heuristics are developed, which are compared to the optimal policy and to each other in Section 6. Finally, Section 7 concludes.

2. Problem formulation

In this section we formulate the problem of shipping and taking back stock for a library system with n libraries and one depot. The depot is indexed by $i = 0$ and the libraries by $i = 1, \ldots, n$. The system is depicted schematically in Fig. 1. A downstream movement of stock from the depot to the libraries is called a *shipment*. An upstream movement from a library to the depot is called a *take-back*.

We consider the inventory control for a single item type, e.g., a specific book title. It seems reasonable to assume that in settings with low demand and quick shipments of back-up stock from a depot, the effect of substitution in case of stock-outs is negligible. Therefore, we can repeat our analysis for every item type in case there are multiple item types. It is straightforward to extend the mathematical model with substitution by including multiple item types, however, given that the problem without substitution is already of significant interest, we believe such an extension is beyond the scope of this paper.

The total number of copies of this item is fixed and given by K. Libraries in practice typically allow a limited number of backorders per library in order to reduce administrative inconvenience and waiting times. We let $B > 0$ be the maximum number of backorders per library; any additional demand is lost. In case there is full backordering, which could be the case for other rental companies, we could set B large enough to approximate full backordering situations. In Fig. 1, $x_{0t} \geq 0$ is the on-hand inventory at the depot in period t. The on-hand inventory at library $i, i = 1, \ldots, n$ is given by $x_{it} \geq -B$. Similarly, $y_{it} \geq 0$ is the number of items rented from library $i, i = 1, \ldots, n$. The state S_t of the system in period t is represented as $S_t = (x_{0t}, x_t, y_t)$, with $x_t = (x_{1t}, \ldots, x_{nt})$ and $y_t = (y_{1t}, \ldots, y_{nt})$.

The library system employs a periodic review policy. In typical library systems, these reviews are executed on a daily, biweekly, or weekly basis. The time line of events is summarized in Fig. 2, where the state after demands/returns, shipments, and take-backs are indexed with zero, one, and two primes, respectively. For example, the on-hand inventory levels after these respective phases are given by x_t, x'_t and x''_t.

Period t starts with clients demanding D_t new items and returning R_t previously rented items at the libraries. The demand and return processes are as follows. Library i faces demand D_{it} during period t. We use the common Poisson process for modeling customer arrivals, hence $D_{it} \sim \text{Poisson}(\lambda_i)$ with λ_i a library spe-

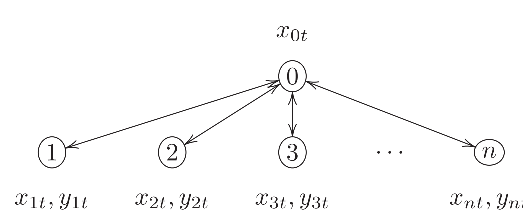

Fig. 1. A library system with n libraries and depot.