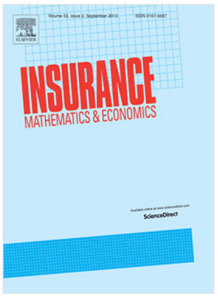
Accepted Manuscript



Distortion measures and homogeneous financial derivatives

John A. Major

PII: S0167-6687(17)30338-4
DOI: https://doi.org/10.1016/j.insmatheco.2017.12.006
Reference: INSUMA 2430

To appear in: *Insurance: Mathematics and Economics*

Received date : July 2017
Revised date : December 2017
Accepted date : 11 December 2017

Please cite this article as: Major J.A., Distortion measures and homogeneous financial derivatives. *Insurance: Mathematics and Economics* (2017), https://doi.org/10.1016/j.insmatheco.2017.12.006

Distortion Measures and Homogeneous Financial Derivatives

John A. Major, ASA

Director of Actuarial Research, Guy Carpenter & Company, LLC

john.a.major@guycarp.com
39 Donna Drive
Burlington, CT, USA 06013

This paper extends the evaluation and allocation of distortion risk measures to apply to arbitrary homogeneous operators ("financial derivatives," e.g. reinsurance recovery) of primitive portfolio elements (e.g. line of business losses). Previous literature argues that the allocation of the portfolio measure to the financial derivative should take the usual special-case form of Aumann-Shapley, being a distortion-weighted "co-measure" expectation. This is taken here as the definition of the "distorted" measure of the derivative "with respect to" the underlying portfolio. Due to homogeneity, the subsequent allocation of the derivative's value to the primitive elements of the portfolio again follows Aumann-Shapley, in the form of the exposure gradient of the distorted measure. However, the gradient in this case is seen to consist of two terms. The first is the familiar distorted expectation of the gradient of the financial derivative with respect to exposure to the element. The second term involves the conditional covariance of the financial derivative with the element. Sufficient conditions for this second term to vanish are provided. A method for estimating the second term in a simulation framework is proposed. Examples are provided.

Keywords: distortion measures, financial derivatives, capital allocation, Aumann-Shapley, reinsurance.

JEL classifications: C71, D81, G22

1. Introduction

This paper discusses the allocation of capital or costs in the particular situation where (1) the capital or costs are computed by a distortion risk measure, (2) that measure is applied to a portfolio of liabilities whose total loss is the sum of the component loss random variables, (3) a nonlinear homogeneous risk transformation (e.g., reinsurance) is contemplated in order to decompose the total loss into a ceded portion that will be paid by another party and the complementary retained portion that will remain in the portfolio, (4) it is desired to evaluate the impact of that risk transformation on capital or costs, and (5) it is desired to allocate that impact back to the original component loss random variables.

Distortion measures are an important class of coherent risk measures. Kusuoka [2001] proved that distortion measures are the only law invariant comonotonic additive coherent risk measures. Distortion