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Thermal response construction in randomly packed solids with graph theoretic support vector regression

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ABSTRACT

Material discontinuities and heterogeneities are very common in engineering systems. The thermal response of such systems to heating and cooling is strongly dependent upon the thermal resistance found at the interfaces. Due to difficulties in modeling thermal resistances between different solid objects of a constituent system or assembly, an alternative approach is proposed in this work. In this paper, the dynamic flow of heat through multiple solid objects within a system is predicted by a machine learning model that uses a graph theoretic diffusion kernel. To demonstrate this approach, a randomly packed assembly of cylindrical rods was selected to represent a network of solids where heat transfer rates are largely governed by contacts and/or gaps between the solids. This assembly of cylindrical rods represents a network of solids with unknown interconnecting thermal resistances. Each rod represents a node in the network. The cylindrical rods were assembled in a quartz tube with axis aligned parallel to each other and the tube. The assembly was heated to temperatures up to 1200 K, and then cool-down experiments were conducted. An infra-red (IR) camera and IR transparent ports in the high temperature experimental facility were used to obtain time dependent, high-fidelity thermographic data for training and validation purposes. In particular, the temperature information from an 'outer' set of rods, i.e., directly cooled by contact with an external surface, was used for generating training data. This data was then used to train the models for predicting the thermal response of 'inner' rods of graphite, quartz and alumina materials. The network of 'inner' and 'outer' rods touching each other was used to construct a graph. A Support Vector Regression (SVR) model was trained using a diffusion kernel obtained from this graph's Laplacian matrix. The regression models were used to predict the thermal response in different materials, sizes and networks. When trained with the optimal thermographic data, the RMS error magnitudes from these model predictions were consistently below 3%.

1. Introduction

Numerous thermal engineering applications – such as nuclear waste systems, nuclear reactors, high temperature catalytic reactors, heat removal from computer chips – involve two major challenges: in situ measurement of temperature at different locations within the spatial domain, and material heterogeneities or discontinuities within the domain that makes their thermal behavior prediction or estimation difficult. These practical examples involve heat transfer across fixed (welded, soldered) or loose joints between solid objects. In these cases, it is often difficult to predict or measure temperature – a problem that makes thermal design particularly challenging. Direct or indirect heat transfer between solids is also of significance in various problems in which the materials (solids) develop cracks and the effective heat transfer through the material is dependent upon the thermal contact conductance between the cracked, discontinuous regions [1].

When these heterogeneous systems have solids in contact with one another, they exhibit finite thermal contact conductance and radiative heat transfer through interacting surfaces. In order to use the classical continuum approach to predict the thermal behavior of such systems, it is important to understand and resolve the thermal contact conductance. Although there are several thermal contact conductance models detailed in the literature [2–4], they assume that contact spots are circular with identical radii. However, in a realistic situation, when two objects are brought in contact, surface irregularities introduce variations in their contacts. Predicting energy flow using these models require detailed information about these contacts and the resulting gaps between the

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Nomenclature

Symbols

θ	temperature
θ_0	temperature of the ghost node
$\mathbf{K}$	diffusion kernel matrix
$\mathbf{L}$	Laplacian matrix
$\mathbf{w}$	directional vector
l_0	weight of the ghost node
t	time
T_0	initialization time
τ	temporal constant
σ	spatial constant
ϵ	insensitive loss function
λ	Lagrange multiplier
b	hyperplane bias

Subscripts

O	outer rods
I	inner rods
i, j	node or rod indices

solids, which is usually very difficult to obtain. Other limitations associated with adopting continuum models for these complex geometries are that analytical solutions do not exist, and numerical discretization becomes challenging as the nodal contacts between different heterogeneous regions, where interface boundary conditions are applied, are completely grid size dependent.

Various models have been proposed in the literature to overcome some of these limitations [5,6]. Yovanovich [6] reviews the role of geometry, mechanics and thermal physics to model thermal contact resistances, and discusses various improvements on the earlier models. Another mechanical, geometrical approach to model thermal contact conductance was proposed by Salgon et al. [7], which depends upon contact area between two bodies. Singhal et al. [8] developed surface topography and material property dependent model for axially contacting cylinders which was experimentally validated. Xu et al. [9] described thermal contact at different roughness scales using a fractal description. However, the requirement that the contacts need to be described at macroscopic to microscopic levels based on resolution of the geometry, still persists. This increases the complexity and reduces the reliability and general applicability of the models. In large systems with length scales of a few meters, it is difficult to resolve features below millimeter level. Verma et al. [10] recently introduced a stochastically reconstructed topography to extract a thermal contact conductance model at interfaces between solids.

Traditionally, thermal contact conductance is modeled as a heat transfer coefficient that is obtained from the experimental data. Thus, thermal contacts arising due to physical discontinuities have previously been modeled using empirical correlations based on experimental data availability. Data obtained from past experiments generally consisted of embedded thermocouple responses. This provided limited information at very low spatial resolution. Thermocouple embedding is associated with creation of heterogeneous contacts between two different materials. Models constructed from these low fidelity experimental data were not sufficient to capture either the spatial or the temporal effects of the individual point contacts. Therefore, uncertainties associated with these intrusive temperature measurement systems are not negligible. Additionally, it may not even be possible to install such intrusive temperature measurement devices in many practical systems. Therefore, learning from experiments using detailed local measurements via non-intrusive instrumentation is probably the most effective approach. Due to the high resolution data requirements for such models, high fidelity non-intrusive techniques were developed [11–13]. A study by Burghold et al. [11] provides a method to estimate heat transfer coefficients with transient temperature measurements using thermographic images. Dynamic estimates of heat transfer mechanisms can then be utilized for describing the transient heat transfer under different test conditions. With improved resolution and speed, modern IR thermography can significantly improve the understanding and modeling of thermal contact resistances in various practical systems. Thermal conduction in homogeneous solids at the macroscopic scale is a well established theory. Thus, if the temperature data at the surface of the solid could be obtained from surface measurements or remote measurements, the internal temperature distribution for homogeneous regions can be computed easily for dynamic or static physics. In the past, several researchers have solved inverse heat conduction problems to predict temperatures or heat flux conditions at unknown locations, using the measured or estimated data at known locations [14–17]. Colaco and Alves [18] developed a non-intrusive inverse heat transfer method using reciprocity functional approach to estimate spatially varying thermal contact heat transfer coefficients.

However, inverse heat transfer problems involving non-homogeneous solids or assemblies of different solids, where multiple thermal contacts and gaps between different solids constitute the major heat transfer resistance, can be daunting task. This is because thermographic or non-intrusive measurements of the temperature at any internal contacts are not possible even through an IR transparent window. The challenge is to use the surface temperature measurements obtained from an IR camera and other limited information such as geometric arrangement in order to construct the internal temperature distributions of multiple solids in thermal or physical contact. A spatio-temporal regression algorithm is required to fuse the surface IR signal response to the material or geometric information in order to obtain the temperature map for the entire domain of interest.

There have been some recent attempts to adopt machine learning models to solve inverse heat transfer problems. However, their scope has been limited to simple geometries with limited physical constraints [19,20]. Additionally, these models are based on the conductive mode of heat transfer, so their application is generally limited to lower temperature regimes where radiative heat transfer can be neglected. In this paper, a novel machine learning approach is introduced to estimate the dynamic thermal responses in complex random arrangements of solids using support vector regression and algebraic graph theory. To illustrate this approach, assemblies of randomly packed, parallel cylinders with length-to-diameter ratios $\gg 1$ were chosen to provide the test geometry. The rationale behind the choice of these assemblies was that the high length-to-diameter ratios of the cylindrical rods allow the system to be considered as a planar (2-D) network, with each cylindrical rod being considered as a node in the network. With this geometry, it is easy to make a simplified approach of categorizing lumped solids as either in or out of direct contact with their neighbors. The chosen assembly geometry was also selected to represent 2-D heat transfer through a network of solids where the rate of heat transfer between the solids is almost entirely determined by their mutual minimal contact rather than by intra-solid