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# Role of regression analysis and variation of rheological data in calculation of pressure drop for sludge pipelines

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ABSTRACT

Sludge pumps in wastewater treatment plants are often oversized due to uncertainty in calculation of pressure drop. This issue costs millions of dollars for industry to purchase and operate the oversized pumps. Besides costs, higher electricity consumption is associated with extra $CO_2$ emission which creates huge environmental impacts. Calculation of pressure drop via current pipe flow theory requires model estimation of flow curve data which depends on regression analysis and also varies with natural variation of rheological data. This study investigates impact of variation of rheological data and regression analysis on variation of pressure drop calculated via current pipe flow theories.

Results compare the variation of calculated pressure drop between different models and regression methods and suggest on the suitability of each method.



## 1. Introduction

Optimisation of pumping systems can significantly reduce the energy consumption by wastewater treatment plants since pumps are among the highest energy consumers in wastewater treatment processes. Rheological characterisation of sludge is important since it provides the essential information used in the design and optimisation of pumping systems (Eshtiaghi et al., 2013; Slatter, 1997).

The pipe flow theory for sludge is well developed (Slatter, 2003, 2008) but the wide variability of the reported rheological model parameters hinders application of the theory to practical engineering design (e.g. the variability in Herschel-Bulkley (HB) model parameters used to describe sludge flow behaviour). This variability is more than an order of magnitude for some of the parameters (Anderson et al., 2008; Water Environment, 2008). The reason for the variability in sludge rheological data is unknown, but there are several possible reasons for this: One is that due to the difference in sludge sourcing and its treatment method, the sludge characteristics vary between sites (Jin et al., 2003). The second possibility is temporal variability in sludge characteristics at a single site. This temporal variability may be a result of the temporal change in sludge composition which occurs with the change of season (Mahmoud et al., 2004; Water Environment, 2008) or time of sampling. Another possibility is the reliability of the method used for the analysis of sludge rheological data (i.e. nonlinear regression) (Ratkovich et al., 2013). This is more important where a three-parameter model such as HB fits the sludge flow curve. This inconsistency has resulted in proposing different relationships for the variation of HB model parameters with sludge concentration (Dabak and Yucel, 1987; Farno et al., 2015; Lotito et al., 1997; Mori et al., 2006).

As of other biological systems (He, 2014; Zi, 2011), not only sludge composition naturally contains variation but also sludge rheological data and model parameters introduce random errors (Dochain and Vanrolleghem, 2001). Regardless of the source of errors, such non-deterministic features in every realistic engineering design contribute to stochasticity in the outcome (Dochain and Vanrolleghem, 2001).

Engineers face two problems in the design of pumping systems for sewage sludge. One is the variability in the pressure drop as a result of the variability in the nature of sewage sludge. Second is the non-Newtonian behaviour of sewage sludge which requires accurate rheological data and not relying on the rule-of-thumb approaches (Anderson et al., 2008; Slatter, 2001). The design of these systems requires a reliable means of estimating pressure drop at various flow rates. This is a complicated task because the

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description of the complex behaviour of sludge (i.e. flow behaviour) typically requires two or three rheological parameters rather than the more traditional use of a single 'viscosity' value for simpler fluids (Malkin et al., 2004; Slatter, 2004).

Municipal sewage sludge is known as a shear-thinning visco-plastic material. Literature has mostly used Bingham Plastic (BP), Herschel-Bulkley (HB) and recently modified Herschel-Bulkley (modified HB) models to describe sludge flow curve:

$$\tau = \tau_B + k\dot{\gamma} \tag{1}$$

$$\tau = \tau_H + k\dot{\gamma}^n \tag{2}$$

$$\tau = \tau_H + k\dot{\gamma}^n + \alpha\dot{\gamma} \tag{3}$$

Where $\tau$ [Pa] is shear stress, $\gamma = -dv/dr$ [$s^{-1}$] is shear rate, $v$ [m/s] is velocity along the flow direction (e.g., along a pipeline), $r$ [m] is flow cross section distance (e.g., radial distant from pipe centre), and $\tau_B$ [Pa], $\tau_H$ [Pa], $k$ [Pa.s$^n$], $n$ [$-$] and $\alpha$ [Pa.s] are fitting parameters of the models.

BP is a linear model which has only two fitting parameters. Applying a linear model results in an easy straight-forward regression approach. However, this model predicts apparent viscosity with high errors at medium and high shear rates as it ignores sludge widely known shear-thinning characteristics (Baudez et al., 2011). In contrast, HB model has three fitting parameters and well fits sludge flow curve at medium and low range shear rates. Addition of a power exponent in Eq. (2) results in a nonlinear model which requires a nonlinear fitting approach. The drawback of using HB model is that it under estimates apparent viscosity at high shear rates (Baudez et al., 2011). This issue is fixed by adding the third term in Eq. (3) compromising modified HB with four fitting parameters. Modified HB model well fits the sludge flow curve at a wider shear-rate range due to a higher number of fitting parameters (Baudez et al., 2011). However, this additional parameter may induce uncertainty over fitting parameters as well as in the calculation of pressure drop in pipeline.

The main purpose of fitting rheological models to sludge flow curve is to predict the pressure difference required to properly transfer sludge in pipe lines (Carthew et al., 1983; Eshtiaghi et al., 2013; Mulbarger et al., 1981). According to the pipe flow theory (Bird, 2002), shear stress along a pipe distributes linearly across the pipe cross-section.

$$\tau = \left(-\frac{\Delta P}{L}\right)\frac{r}{2} \tag{4}$$

Where $\Delta P$ [Pa] is the pressure difference along a pipe with the length of $L$ [m].

However, sludge has a yield stress (e.g., $\tau_B$, $\tau_H$) below which it does not flow. As a result, close to the pipe centre and depending on the imposed pressure difference, there might be an area in which the actual shear stress which the sludge experiences is less than its yield stress. In this case, there will be a solid plug-like core in the middle of the pipe. In other words, if the pressure difference imposed on sludge to flow in a pipeline is not large enough, part of the sludge remains unsheared and plug flow will form. The radius of plug flow is calculated as follows:

$$R_p = \left(\frac{\tau_y}{\tau_w}\right)R \tag{5}$$

where $R$ [m] is pipe radius, $\tau_y$ [Pa] is yield stress and $\tau_w$ [Pa] is shear stress at pipe wall (Chhabra and Richardson, 2011).

If it is assumed that the model yield stress is equal to actual material yield stress ($\tau_y = \tau_H$), then in the laminar regime, velocity is calculated by integrating Eq. (2) in respect to $r$ and satisfying no wall slip condition ($v_{\text{ at } r = R} = 0$).

$$v = \left(\frac{nR}{(n+1)}\right)\left(\frac{\tau_w}{k}\right)^{\frac{1}{n}}\left(\left(1-\frac{\tau_H}{\tau_w}\right)^{\frac{n+1}{n}} - \left(\frac{r}{R}-\frac{\tau_H}{\tau_w}\right)^{\frac{n+1}{n}}\right) \; r > Rp \tag{6}$$

$$v = \left(\frac{nR}{(n+1)}\right)\left(\frac{\tau_w}{k}\right)^{\frac{1}{n}}\left(1-\frac{\tau_H}{\tau_w}\right)^{\frac{n+1}{n}} \; r < Rp \tag{7}$$

Flow rate is also calculated by integrating velocity across the pipe cross-section:

$$Q = \pi R^3 n\left(\frac{\tau_w}{k}\right)\left(1-\frac{\tau_H}{\tau_w}\right)^{(n+1)/n}\left(\frac{\left(1-\tau_H/\tau_w\right)^2}{3n+1} + \frac{\left(2\tau_H/\tau_w\right)\left(1-\frac{\tau_H}{\tau_w}\right)}{2n+1} + \frac{\left(\tau_H/\tau_w\right)^2}{n+1}\right) \tag{8}$$

Substituting $n = 1$ in Eqs. (6)–(8) yields velocity and flow rate for BP model (Eq. (1)) instead of HB model (Eq. (2)). However, substituting Eq. (4) in modified HB model (Eq. (3)) yields the following nonlinear ordinary differential equation which, to our best knowledge, no analytical solution has yet been developed for.

$$\left(-\Delta P/_{L}\right) r/2 = \tau_H + k\left(-dv/_{dr}\right)^n + \alpha\left(-dv/_{dr}\right) \tag{9}$$

In this study, a numerical solution was developed in MATLAB to calculate velocity and flow rate based on modified HB model.

Eqs. (6)–(8) and numerical solution of Eq. (9) are only valid in laminar regime. In turbulent regime, no exact mathematical analysis has yet been developed for HB or BP. Several semi-theoretical equations have been proposed based on modifications of expression of shear stress at the pipe wall for Newtonian fluids (Chhabra and Richardson, 2011; Slatter, 2008). Slatter (2008) proposed a logarithmic velocity distribution for turbulent flow of non-Newtonian fluids as follows:

$$v/_{v_*} = 2.5\ln\left(\frac{r}{\varepsilon}\right) + B - 3.75 \tag{10}$$

Where $v_* = (\tau_{y/\rho})^{1/2}$, $\rho$ [kg/m$^3$] is fluid density, $\varepsilon$ is pipe roughness and $B$ is classic roughness function.

In Slatter (2008) approach, $B$ varies with rough wall Reynolds number defined as follows:

$$Re_r = \frac{8\rho v_*^2}{\tau_y + k\left(8v_*^2/_{\varepsilon}\right)} \tag{11}$$

For concentrated sludge, Slatter (2008) showed $B$ equals to $8.5$ at $Re_r > 3.32$ (rough wall turbulent flow) and $B$ equals to $2.5\,Ln(Re_r) + 5.5$ at $Re_r \le 3.32$.

Therefore, the flow regime needs to be identified to know which equation (either Eqs. (6)–(9) or Eq. (10)) is applied.

In flow of Newtonian fluids in pipe, the flow regime changes from laminar to turbulent regime at the Reynolds number of *2100*. Based on the original definition of Reynolds number, Slatter (1995) formulated $Re_3$ (Eq. (12)) for fluids that obey HB and BP models (Reynolds number defined as the ratio of inertial to viscous forces is used to identify the flow regime). In $Re_3$ approach, the transition of regime from laminar to turbulence also occurs at *2100*.