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A theorem for calculation of the appropriate sample size in an estimation

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ABSTRACT

In 1982, Dubois and Prade investigated the relationship between belief function, plausibility function and basic probability assignment when the involved universe is finite. In this paper, the similar results on their relationships are obtained with a continuous universe. As an important facility to connect possibility distribution in continuous universes and discrete probability values, basic probability histogram is defined by means of measurement amplitude, which is a notion with both probability and possibility features. A theorem about how to calculate a suitable sample size for estimation is proposed based on the researches on basic probability histograms. Through the theorem, we can directly calculate the appropriate samples size for any population distribution. Even with small samples, a reasonable estimation can be obtained with a non-normal distribution.

1. Introduction

Many new perspectives applying to dealing with uncertainty issues have been proposed since Zadeh put forward the notion of fuzzy set [1]. Powerful advantages could be seen in the theories such as Dempster-Shafer theory[2] and possibility theory[3] which are used to solve uncertainty issues and they are regarded as comparable with probability theory.There are two general problems that we must face when a fuzzy decision is made by using these theories. One is about constructing a fuzzy membership function from statistical data, the other is about selecting a suitable sample size for estimation. In this paper, we aim to present a finding on an effective calculation method to give a suitable sample size for using a random sample to estimate the target accurately. Since samples are probable while the overall estimation process is plausible, the former is essentially a problem on the conversion between possibility and probability. The following is a brief introduction to the conversion problem and the sample size problem.

1.1. The conversion between possibility and probability

The reason why the conversion problem between probability and possibility had received much attention in the past is that the transformation is useful in some practical problems, e.g. in constructing a fuzzy membership function from statistical data [4], combining probabilities and possibilities in expert systems [5] and reducing complicated complexity [6]. This conversion problem is rooted in possibility-probability consistency principle which was introduced by Zadeh, in his paper founding possibility theory [3]. He pointed out that there is a heuristic connection between possibilities and probabilities, i.e. a high degree of possibility does not imply a high degree of probability, nor does a low degree of probability imply a low degree of possibility. However, if an event is impossible, it is bound to be improbable. In 1982, a further study on the existing axiomatic theories, Shafer's belief theory [2] and the triangular norm-based approach [7,8], dealing with large classes of fuzzy measures was done by Dubois and Prade [9,10]. Their study shows that from both approaches emerge three remarkable families of fuzzy measures: the probability, possibility and necessity measures. They pointed out that the possibility measure is a particular case of plausibility functions while necessity measure is a particular case of belief functions. And they proved that Shafer's belief and plausibility functions can be represented via a basic probability assignment (which is nothing but a random set), and also that probability measure, possibility measure and necessity measure can be expressed in terms of a density. They suggested regarding this feature as a general framework for the combination of uncertain information. Afterwards, many researchers, such as Wang [11], Klir [12], Dubois and Prade [6], Baruah [13,14] etc., have conducted useful research on the conversion problem

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between probability and possibility. Although there exist many viewpoints that are not consistent with each other,it seems very reasonable to regard the general framework introduced by Dubois and Prade as a guiding principle for studying of the consistency relationship existing between possibilistic and probabilistic. Another important thing to be noted here is that most of consistency principles that can be found in the literature of fuzzy set theory deal with the consistency in discrete cases. Continuous cases were discussed a bit in a paper of Dubois-Prade-Shandri [6], which involved the transformation from possibility measure to probability measure T1 $(T1(\pi)(x) : p(x) = T1(\pi)(x) = \sum_{i=1}^{n} \frac{\pi_i - \pi_{i+1}}{|A_i|} \mu_{A_i(x)}$. where $\pi(x)$ is a possibility density function in *D;* $A_1, \cdots, A_n$ is a finite set of level-cuts corressponding to $\pi_1 = 1 > \pi_2 > \cdots > \pi_n > \pi_{n+1} = 0$; $|A_i|$ is the cardinal of set A_i; $\mu_{A_i}(x)$ is the characteristic function of A_i) and the transformation from probability measure to possibility measure T2 $(T2(p) : \pi(x) = \pi(f(x)) = \int_{-\infty}^{x} p(y)dy + \int_{f(x)}^{+\infty} p(y)dy$. where p is a probability density function in *D. f* is a function define as *f*: $[a, x_0] \to [x_0, b]$ by $f(x) = max(y|p(y) \geq p(x))$, π is the possibility distribution .) But at the same time it was mentioned that these two transformations were not related to each other and the converse transformations were shown to be inadequate. In this paper, we show that the framework introduced by Dubois and Prade can be extended to continuous universes and has almost the same expression and relationship as in discrete universes.

1.2. The sample size for estimation

Deducing the features of the population from the sample data is one basic task of statistics. Sample size influences the credibility of the estimation results directly. Modern Statistics indicates that bigger sample size brings the higher credibility of the estimation results [15–17]. However, it usually costs a large quantity of manpower and material resources to obtain large samples. As a result, the study of appropriate sample size not only ensures the estimation results to meet the needs of the following work but also consider the cost in the process of collecting sample data. Since the establishment of fuzzy set, the authors have not found literatures that use fuzzy theory to study the sample size problem in fuzzy decision making. Most of the literatures about calculating sample size employ chebyshev's theorem or hypothesis test theory of statistics [18,19]. In statistics, the sample population of hypothesis test is required to be subject to the normal distribution [20]. The non-normal distribution usually need to be normalized by the central limit theorem. But the central limit theorem is established with the condition that the sample size n is sufficiently large [21,22]. The smaller sample size will lead to the failure of getting an ideal statistic conclusion [23,24]. In this paper, based on the a measurement experiment, a theorem about how to calculate a suitable sample size for estimation is proposed. Through the theorem, we can directly calculate the appropriate samples size for any population distribution. Even with small samples, a reasonable estimation can be obtained with a non-normal distribution.

The rest of the paper has been organized as follows: Section 2 recalls main conclusions of the framework introduced by Dubois and Prade. Section 3 extends the results of Dubois and Prade 1982 to continuous universes by two important concepts which are defined by the authors: measurement amplitude and basic probability histogram. Section 4 introduces an approach to construct the credibility distribution of estimation. Section 5 provides a theorem on how to calculate the appropriate sample size for estimation. Finally, Section 6 is our conclusions.

2. The connection of plausibility function, belief function and basic probability assignment in the discrete universes

On finite universe X which is the basis for the study of Dubois and Prade, a belief function is a set function *Bel* from $\wp(X)$ (power set of X) to [0,1] such that 1) $Bel(\phi) = 0$; 2) $Bel(X) = 1$; 3) $\forall n \in N, \forall A_i \subset X, i = 1, \cdots, n$,

$$Bel\left(\bigcup_{i=1}^{n} A_i\right) \geq \sum_{k=1}^{n} \left[(-1)^{i+1} \sum_{1 \leq i_1 < \cdots < i_k \leq n} Bel\left(A_{i_1} \bigcap \cdots \bigcap A_{i_k}\right) \right]. \tag{2.1}$$

When n=2, it yields the following important inequalities

$$\forall A, B \subset X, Bel\left(A \bigcup B\right) \geq Bel(A) + Bel(B) - Bel\left(A \bigcap B\right). \tag{2.2}$$

$$\forall A, B \subset X, Bel\left(A \bigcap B\right) \geq \max\left(0, Bel(A) + Bel(B) - 1\right). \tag{2.3}$$

$$\forall A, B \subset X, Bel\left(A \bigcap B\right) \leq \min\left(Bel(A), Bel(B)\right). \tag{2.4}$$

$$\forall A \subset X, Bel(A) + Bel(\bar{A}) \leq 1, \tag{2.5}$$

where $\bar{A}$ is the complement of A.

The set function *Pl* named plausibility function dual to belief function is defined by

$$\forall A \subset X, Pl(A) = 1 - Bel(\bar{A}). \tag{2.6}$$

There are also four inequalities dual to (2.2) through (2.5)

$$\forall A, B \subset X, Pl\left(A \bigcap B\right) \leq Pl(A) + Pl(B) - Pl\left(A \bigcup B\right). \tag{2.7}$$

$$\forall A, B \subset X, Pl\left(A \bigcup B\right) \leq \min\left(1, Pl(A) + Pl(B)\right). \tag{2.8}$$

$$\forall A, B \subset X, Pl\left(A \bigcup B\right) \geq \max\left(Pl(A), Pl(B)\right). \tag{2.9}$$

$$\forall A \subset X, Pl(A) + Pl(\bar{A}) \geq 1. \tag{2.10}$$

The following inequality holds

$$\forall A \subset X, Pl(A) \geq Bel(A). \tag{2.11}$$

The basic probability assignment is a mapping m from $\wp(X)$ to [0, 1] such that 1) $m(\phi) = 0$; 2) $\sum_{A \subset X} m(A) = 1$. Through a basic probability assignment, the belief function and the plausibility function can be respectively expressed as

$$\forall A \subset X, Bel(A) = \sum_{B \subset A} m(B). \tag{2.12}$$

$$\forall A \subset X, Pl(A) = \sum_{B \bigcap A \neq \phi} m(B). \tag{2.13}$$

It is shown that m can be obtained from *Bel* by

$$\forall A \subset X, m(A) = \sum_{B \subset A} (-1)^{|A-B|} Bel(B), \tag{2.14}$$

where $|A|$ denotes the cardinality of the set A.