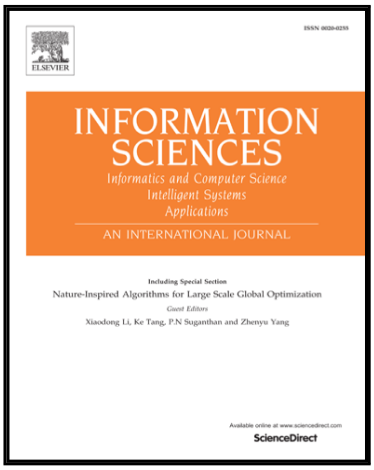

The distributive laws of fuzzy implications over overlap and grouping functions

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The distributive laws of fuzzy implications over overlap and grouping functions

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Abstract

Overlap and grouping functions, as two kinds of special binary aggregation functions, have been investigated for applications in image processing, classification problems and decision making based on fuzzy preference relations. In addition, after the distributive laws related to fuzzy implications and some special binary aggregation functions have been proposed, there arise many discussions on this research topic. In this paper, we continue studying this topic and discuss the four basic distributive laws of fuzzy implications over overlap and grouping functions. At first, we investigate the four basic distributive laws of fuzzy implications over overlap and grouping functions for the residual implications derived from overlap functions, (G, N)-implications obtained from grouping functions G and fuzzy negations N, and QL-operations derived from overlap functions, grouping functions and fuzzy negations, respectively. And then, we extend the related results when we take into account the residual implications, (G, N)-implications or QL-operations in the four basic distributive laws of fuzzy implications over overlap and grouping functions to arbitrary fuzzy implications satisfying some desirable algebraic properties in the preceding four basic distributive laws.

Keywords: Overlap functions; Grouping functions; Fuzzy implications; Distributive laws

1. Introduction and motivation

1.1. A brief review on overlap and grouping functions

Overlap and grouping functions [16, 18] are introduced by Bustince et al. in 2009 and 2012, respectively. These two concepts, as specific nonassociative binary aggregation functions, arise from some problems in image processing [15], classification [30] or decision making [47], where t-norms and t-conorms are commonly used [32]. But nonetheless, as a matter of fact, the associativity of t-norms and t-conorms is not forcefully needed in many cases.

In recent years, overlap and grouping functions have seen a rapid development both in applications and theory. In applications, overlap and grouping functions play a vital role in image processing, decision making and classification problems. For example, in image processing, in 2010, Bustince et al. [17] proposed an object recognition problem in which the best classification with respect to the background is the one with less overlapping between the class background and the class object. In 2013, Jurio et al. [34] obtained that the convex combination of several overlap functions (resp. grouping functions) is again an overlap function (resp. a grouping function). In addition, they showed the advantages of this kind of consensus by presenting an application in image processing. In decision making, in 2012, Bustince et al. [18] elaborated the use of overlap and grouping functions in fuzzy preference modeling and decision making. They also presented an algorithm to elaborate an alternative preference ranking that penalizes the alternatives for which the expert is not sure about his or her preference. In classification, in 2015, Elkano et al. [30] adapted the inference system of fuzzy association rule-based classification model for high-dimensional problems by replacing the product t-norm with n-dimensional overlap functions, which allows one to obtain more adequate outputs from the base classifiers for the subsequent aggregation in One-versus-One and One-versus-All schemes. In the same year, Lucca et al. [37] introduced one class of Choquet-based non-associative aggregation functions for application in

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