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# Stability analysis methods and their applicability to car-following models in conventional and connected environments

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ABSTRACT

The paper comprehensively reviews major methods for analysing local and string stability of car-following (CF) models. Specifically, three types of CF models are considered: basic, time-delayed, and multi-anticipative/cooperative CF models. For each type, notable methods in the literature for analysing its local stability and string stability have been reviewed in detail, including the characteristic equation based method (e.g., root extracting, the root locus method, the Routh–Hurwitz criterion, the Nyquist criterion and the Hopf bifurcation method), Lyapunov criterion, the direct transfer function based method, and the Laplace transform based method. In addition, consistency and applicability of stability criteria obtained using some of these methods are objectively compared with the simulation result from a series of numerical experiments. Finally, issues, challenges, and research needs of CF models' stability analysis in the era of connected and autonomous vehicles are discussed.



## 1. Introduction

As a typical feature of traffic congestion, stop/slow-and-go oscillations are a main negative externality of road transportation systems, often triggered by instability in the car following (CF) behaviour (Chandler et al., 1958; Wilson and Ward, 2011; Zheng et al., 2011). The growth of perturbation (e.g. a spacing and speed deviation from a steady state) over time and space during CF leads to the instability of traffic flow. In general, the aim of stability analysis is to study how the perturbation (perturbation and disturbance are used interchangeably in this paper) of a leading vehicle evolves over time and space using CF models by assuming that vehicles travel on a single lane without overtaking.

Broadly speaking, there are two types of stability analysis: linear stability analysis and nonlinear stability analysis. Linear stability analysis focuses on stability characteristics of a system under the influence of a small perturbation, while nonlinear stability analysis on stability characteristics of a system under the influence of a large perturbation. For two main reasons, linear stability analysis has been the theme of the majority of the literature in traffic flow theories where nonlinear CF system is often linearized around the equilibrium point: i) for road traffic, disturbances experienced by road users are often small; and ii) nonlinear stability analysis is much more complicated than linear stability analysis. Thus, linear stability

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analysis is the focus of this paper, too. Before any further discussion, some confusion on terminologies used in the literature needs to be clarified. The fact that stability analysis is an important topic, and thus has been investigated in different disciplines (e.g., numerical analysis, control theory, dynamic systems, and of course traffic flow modelling), has caused confusions in the literature in terms of terminologies. For example, different disciplines may define/classify stabilities differently, as pointed out in Zhang and Jarrett (1997) and Treiber and Kesting (2013). In dynamic systems, only local stability is studied, which is categorized as Lyapunov stability (any sufficiently small initial perturbation always remains small) and asymptotic stability (any sufficiently small initial perturbation tends to zero as time approaches infinity). In traffic flow modelling, two types of stability have also been investigated but defined differently: local stability is to investigate the stability of a single vehicle's movement over time under the influence of a small perturbation that is often originated from the leading vehicle's movement (It is locally stable if the perturbation diminishes over time, which corresponds to asymptotic stability in dynamic systems), while asymptotic stability focuses on the stability of a platoon of vehicles over space under the influence of a small perturbation that is originated from the first vehicle of the platoon (it is asymptotically stable if the perturbation strictly diminishes over space/vehicles). Some researchers in traffic flow modelling also call these two types of stability analysis as single vehicle stability (or platoon stability/plant stability), and stability over vehicles (or string stability), respectively. For more information on different types of stability, see Wilson and Ward (2011) and Treiber and Kesting (2013). In the rest of this paper, unless stated otherwise they are referred to as local stability and string stability, respectively.

Meanwhile, as a core component of traffic flow theories, numerous CF models have been developed in the literature to realistically describe longitudinal vehicular interactions from various perspectives. See (Brackstone and McDonald, 1999; Saifuzzaman and Zheng, 2014; Toledo, 2007) for reviews on CF models. For the convenience of discussion, these CF models are grouped into three categories in this paper: basic CF models (B-CF), time-delayed CF models (TD-CF), and multi-anticipative/cooperative CF models (MAC–CF), as briefly defined below.

- B-CF models: this type of CF models describes longitudinal vehicular interactions between two consecutive vehicles without considering any time delays.
- TD-CF models: compared with B-CF models, this type of CF models considers time delay related to CF.
- MAC–CF models: this type of CF models considers influence of more than one leading vehicle on CF behaviour with or without time delays.

Researchers started implementing stability analysis in CF modelling approximately 60 years ago. Generally, stability analysis in each category of CF models has different requirements and challenges. Although there has been significant progress with various methodologies existing for stability analysis of CF models, as the concept of stability is from the control theory and many methods for stability analysis are developed in other disciplines and usually mathematically heavy, it seems that the traffic flow community is, by and large, still not familiar with many of these methods. A general framework for stability analysis of B-CF has been proposed in Wilson and Ward (2011) to facilitate an easier implementation of stability analysis for researchers in traffic flow. However, a comprehensive review of CF stability analysis studies and a detailed description and comparison of major stability analysis methods are still missing, despite its great need. Particularly, CF model stability analysis' practical implications are largely ignored, and no studies have assessed the consistency between the theoretical criteria (from these stability analysis methods) and the simulation results, and in turn the applicability of the stability criteria obtained from various methods. Moreover, the advent of connected and autonomous vehicles (CAVs) can positively or negatively affect traffic flow's stability (Talebpour and Mahmassani, 2016), which is likely to bring numerous opportunities and challenges of utilising stability analysis and developing effective control strategies accordingly by taking advantage of the connected environment . This further underscores the importance and urgency of revisiting and comparing major stability analysis methods, and assessing their applicability to guide future research related to the stability of traffic flow consisting of traditional, connected, and autonomous vehicles.

This paper fills this gap. For the sake of clarity and focus, the paper concentrates on commonly-used methods for linear stability analysis of representative CF models, rather than attempting to be exhaustive. For each type of CF models, notable methods in the literature for analysing its local stability and string stability are reviewed in detail, including the characteristic equation based method (e.g., root extracting, the root locus method, the Routh-Hurwitz criterion, the Nyquist criterion and the Hopf bifurcation method), Lyapunov criterion, the direct transfer function based method, and the Laplace transform based method. In addition, consistency and applicability of stability criteria obtained using many of these methods are objectively compared with the simulation result from a series of numerical experiments. Particularly, the impact of connectivity and its interaction with time delay on stability is also investigated. Finally, issues, challenges, and research needs of CF models' stability analysis in the era of connected and autonomous vehicles are discussed.

The remainder of this paper is organized as follows: Section 2, 3, and 4 review and compare major stability analysis methods for B-CF, TD-CF, and MAC–CF models in the literature, respectively; Section 5 discusses strengths, issues, challenges, and research needs of CF models' stability analysis; and Section 6 summaries main findings of this study.

## 2. Stability analysis of B-CF models

This section reviews major methods for both local stability and string stability analysis of B-CF models.