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Sensitivity analysis and sensitivity-based design for linear alarm filters☆

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ABSTRACT

This paper conducts sensitivity analysis and sensitivity-based design for linear filter alarm monitoring systems. Based on a derivative-based local sensitivity measure, models are proposed to assess the sensitivity of the system detection errors to changes in the trip point and to uncertainties in the collected data. Then, analytical expressions are derived to quantitatively evaluate the sensitivity of a general linear alarm filter with unknown data distributions. Subsequently, a new sensitivity-based linear filter design method is formulated to minimize a weighted sum of the detection errors subject to upper bounds on the system sensitivities. Extensive simulations with both Gaussian and industrial data are conducted to verify the analytical results and to show trade-offs between the detection errors and sensitivities of linear filter alarm system.

1. Introduction

With increasingly high requirements on safety, reliability and efficiency in industrial plants, the design of effective alarm monitoring systems has become a heated topic during the last decade. Generally speaking, to improve the accuracy of alarm monitoring systems is to reduce the two types of errors: the false alarm rate (FAR) and the miss alarm rate (MAR) (Adnan, Izadi, & Chen, 2011). There are two main research directions related with this error reduction task. The first one is the design of alarm triggering mechanism, including delay-timers (Zang, Yang, & Huang, 2015), dead-bands (Naghoosi, Izadi, & Chen, 2011) and filters (Cheng, Izadi, & Chen, 2011). The second direction is to analyze the connections among alarms to eliminate sequential alarms and ascertain rationalization suggestions, using correlation analysis (Yang, Shah, Xiao, & Chen, 2012), similarity analysis (Ahmed, Izadi, Chen, Joe, & Burton, 2013) and sequence pattern analysis (Lai & Chen, 2015).

Due to the heavy computation burden and implementation cost in the second direction above, research topics on improved alarm triggering mechanism, especially the advanced filter design with a level-crossing alarm generation mechanism, are still among the most promising ones to reduce nuisance and false alarms. Moreover, linear alarm filters are of particular interest since they are not only easy to implement but also effective in performance enhancement. In the literature, firstly, moving average (MA) filters and moving variance filters were designed for known Gaussian distributions in Izadi, Shah, Shook, Kondaveeti, and Chen (2009). Later, a general optimal design framework for linear and quadratic alarm filters (Cheng et al., 2011) was proposed to tackle data with other distributions. Based on the design framework in Cheng et al. (2011), MA filters were analytically proved to be optimal among linear filters under log-concave and symmetric data distributions (Cheng, Izadi, & Chen, 2013).

The majority of existing results for linear alarm filters still rely on the assumption that the trip point and the probability density functions (PDFs) of data under normal and abnormal conditions are known precisely. However, in reality, uncertainties and changes are inevitable in both the trip point and the PDF estimation. Specifically, computational errors exist in the trip point setting. For example, in the linear filter design (Cheng et al., 2013), the solution is found with iterative numerical methods (Cheng et al., 2011), which offer an approximate solution instead of the exactly optimal one due to limited precisions and limited processing power. As for the uncertainties in the PDFs, strictly speaking, the exact PDFs of process signals are unknown. The PDFs are actually estimated based on the collected data and pre-assumed probability models. But measurement noise in data collection is ubiquitous. Uncertainties in both the trip point and the PDFs will propagate into filter design and alarm performance evaluation. Thus, to understand how uncertainties affect the performance of an alarm system, it is important to quantitatively measure the sensitivity of the system behavior with respect to the trip point change and the PDF variations.

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In the literature, sensitivity measures can be generally classified into two types: local sensitivity measures and global sensitivity measures (Borgonovo & Plischke, 2016). Global sensitivity measures are usually obtained using statistical properties (such as variance ratio Borgonovo & Plischke, 2016) based on assigned known density distributions (Saltelli, Ratto, Andres, Campolongo, Cariboni et al., 2008). On the other hand, pre-given PDFs are unnecessary for local sensitivity measures (Borgonovo & Plischke, 2016). Besides, the two types of measures have different applications. Local sensitivity measures are widely used in evaluating output variations due to slight changes in inputs, while global ones deal with input variations over their entire ranges of interest. With the help of proper sensitivity measures, a sensitivity-based design problem can be formulated to meet diverse engineering needs. For example, in mechanical engineering, derivative-based local sensitivity measures are used to optimize the feasibility of a welded beam in Gunawan and Azarm (2004). In control engineering, the sensitivity of a control system to disturbances is usually reflected by a global sensitivity measure namely the sensitivity transfer function. For example, the weighted $\mathcal{H}_\infty$-norm of the sensitivity function was minimized to achieve optimal robust feedback controller in Zames and Francis (1983), and the 2-norm of the sensitivity function quantified the robustness of the assigned pole to parameter perturbations in a dynamical system in Abdelaziz (2015).

In this work, a local sensitivity measure is chosen instead of global ones. Because results obtained with global sensitivity measures highly rely on pre-assumed and limited types of distribution models; without known distributions, the calculation of global sensitivity measures is mathematically troublesome, if not impossible. However, distributions of the collected data are usually unknown and have relatively large diversity for different processes. Moreover, among existing local sensitivity measures, the derivative-based elasticity measure (Borgonovo & Plischke, 2016; Karnavas, Sanchez, & Bahill, 1993) is adopted and customized for linear alarm filters. Specifically, the inputs of the elasticity are chosen as the trip point variation and the PDF offsets caused by noise in the collected data. And the Kullback–Leibler divergence (KLD), a popular distance measure between probability distributions in information theory and statistics (Kullback & Leibler, 1951), is applied to measure the PDF offsets between the ideal noise-free data and the collected noisy data. The MAR and FAR are chosen as the outputs. Without any knowledge on the collected data distribution, sensitivities of the detection errors to the trip point and KLD are calculated based on the Gaussian kernel estimation.

In addition, based on derived sensitivity expressions, a new linear filter design method is proposed, which takes both detection errors and sensitivity measures into account. The design procedure is modeled as a constrained minimization problem, where the objective is to minimize the weighted summation of detection errors within sensitivity constraints. A grid search method is used to locate the solution. Simulations on the Gaussian and industrial data are conducted to verify the effectiveness of the proposed sensitivity analysis and sensitivity-based design. Also, simulation results provide insightful observations in detection errors and the sensitivity of linear alarm filters.

The rest of the paper is organized as follows. Backgrounds on alarm monitoring with linear filters and sensitivity analysis models are illustrated in Section 2. Then in Section 3, the proposed sensitivity models and the detailed calculation procedures are presented, together with the sensitivity-based linear filter design. In Section 4, simulation results on data sets from Gaussian distribution and from an industrial plant are presented and studied. Section 5 concludes this paper.

2. Linear alarm filters and sensitivity models

2.1. Linear alarm filters

Let $\boldsymbol{x} = [x(1), x(2), \ldots]$ be a vector of discrete-time signals. A filter is essentially an operator that processes $\boldsymbol{x}$ to produce another vector discrete-time signals $\boldsymbol{y} = [y(1), y(2), \ldots]$, written as $\boldsymbol{y} = F(\boldsymbol{x})$. In the alarm monitoring area, linear filters show significant advantages over non-linear filters (Cheng et al., 2011) in easy application and simple computation. Therefore, linear filters are considered in this work, with the following form:

$$y(k) = \sum_{i=0}^{N-1} \theta_i x(k-i), \tag{1}$$

where N is the length of the linear filter and θ_i's are the filter coefficients. Without loss of generality, the summation of all the filter coefficients is set to be 1, i.e., $\sum_{i=0}^{N-1} \theta_i = 1$. With the typical level crossing alarm generation mechanism (Izadi et al., 2009), the filtered signal $y(k)$ is compared with the trip point y_{tp} to trigger an alarm, i.e., an alarm will be raised if $y(k) > y_{tp}$.

In this work, collected data under the normal and abnormal modes are denoted as $\boldsymbol{x}_n = [a_1, a_2, \ldots, a_{l_n}]$ with length l_n and $\boldsymbol{x}_{ab} = [b_1, b_2, \ldots, b_{l_{ab}}]$ with length l_{ab} respectively. It is assumed that a_i's are independent and identically distributed (i.i.d.), b_j's are i.i.d., and a_i's are independent to b_j's. Let $f_{X,n}(\cdot)$ and $f_{X,ab}(\cdot)$ be the PDFs of a_i and b_j respectively. Also, let $\boldsymbol{y}_n = F(\boldsymbol{x}_n)$ and $\boldsymbol{y}_{ab} = F(\boldsymbol{x}_{ab})$ denote the filtered signals, and $f_{Y,n}(\cdot)$ and $f_{Y,ab}(\cdot)$ represent the common PDF of each element in $\boldsymbol{y}_n$ and $\boldsymbol{y}_{ab}$ respectively.

The FAR (MAR) of the alarm system is defined as the probability that an alarm is raised (missed) when the system is under the normal (abnormal) mode. The goal in conventional linear filter design is to find the optimal combination of filter coefficients and the trip point with the lowest weighted sum of FAR and MAR. With the trip point y_{tp}, the cost function can be calculated as:

$$\begin{aligned} \mathrm{J_{conv}} &= c_1 \mathrm{FAR} + c_2 \mathrm{MAR} \\ &= c_1 \int_{y_{tp}}^{+\infty} f_{Y,n}(y)\mathrm{d}y + c_2 \int_{-\infty}^{y_{tp}} f_{Y,ab}(y)\mathrm{d}y, \end{aligned} \tag{2}$$

where c_1 and c_2 are positive weights on the FAR and MAR. From (2), it is obvious that the FAR, the MAR and the cost function can be significantly affected by the accuracy of the data PDFs and the trip point. Uncertainties in the estimated PDFs and the trip point will lead to performance change. Thus, there exists a strong need to quantify the sensitivity to these changes.

2.2. Sensitivity definition and properties

In this work, the derivative-based elasticity, a local sensitivity measure, is adopted because it eliminates unit differences of input and output variations (Borgonovo & Apostolakis, 2001). More rationale for this measure can be found in Borgonovo and Plischke (2016). The measure is defined as:

$$S_O^I = \lim_{\Delta I \to 0} \frac{\Delta O}{O} \bigg/ \frac{\Delta I}{I} = \frac{\mathrm{d}O}{\mathrm{d}I}\frac{I}{O}, \tag{3}$$

where O stands for the output of the sensitivity model and I represents the input with uncertainties. The ratio I/O is used to scale the changes in the input and output over their current values to obtain a normalized dimensionless value. S_O^I is naturally a sensitivity measure of O to I representing how much the output O changes with the input I.

To complete the sensitivity modeling, it is necessary to specify inputs and outputs that are not only relevant but also capable of representing practical industrial needs. According to Borgonovo and Plischke (2016), model outputs should be variables of most interests to the decision maker. So the performance indices, the FAR and MAR, are chosen as outputs for sensitivity analysis. As for the inputs, once the filter design is completed, candidates are the trip point and the estimated PDFs.

The sensitivity of the detection error over the trip point is straightforward to model by using the definition in (3):