Mechanical Systems and Signal Processing 110 (2018) 590–610

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Scaling laws obtained from a sensitivity analysis and applied to thin vibrating structures

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A R T I C L E   I N F O

Article history:
Received 22 August 2017
Received in revised form 14 March 2018
Accepted 15 March 2018

Keywords:
Scaling laws
Sensitivity analysis
Similitude analysis
Sensitivity-based scaling laws
Mechanical vibrations
Vibrating thin structures

A B S T R A C T

Scaling laws are used to scale-up calculation or measurement results from a prototype structure to those of an original structure or vice versa. The scaling laws of mechanical structures are usually obtained from their equations of motion or from a dimensional analysis. This paper proposes scaling laws that are represented by a power law. The power law is directly deduced from the Π-theorem of similitude theory and the coefficients of the power law are obtained from first order sensitivities (sensitivity-based scaling laws). Among existing similitude analysis methods such as dimensional analysis the sensitivity-based scaling laws can be directly obtained from a model without a priori knowledge of its scaling behavior, e.g., from a finite element (FE) model. The applicability of the sensitivity-based scaling laws is demonstrated in three case studies. A simply supported rectangular plate subject to mechanical vibrations is modeled by an analytical approach based on the KIRCHHOFF plate theory and by an FE model based on the MINDLIN-REISSNER theory. The plate's dimensions are considered as design parameters for the sensitivity-based scaling laws. It is found in the first case study that the sensitivity-based scaling laws exactly predict the natural frequencies and the mean squared transfer admittances of scaled models from those of the original plate. This is in agreement with current literature and, thus, the method proposed in this paper can be considered verified. In the second case study the sensitivity-based scaling laws are directly obtained from the FE model. A good accuracy of the predicted vibration responses of the scaled models is found since (1) the scaled model and the original one are in similitude, i.e., the mode shape order and the damping are kept, and (2) the scaled plate can still be considered thin. The latter is assessed by the length (or width) to thickness ratios of the investigated plates and a lower bound is proposed. The third case study comprises a simplified car undercarriage to demonstrate the applicability of the method in practical design engineering. Besides the geometry parameters the material properties are incorporated in the sensitivity-based scaling laws as well. It is found that the natural frequencies as well as the vibration velocities at an arbitrarily chosen receiver point of the scaled models can be accurately predicted from those of the original one for complete similitude conditions.

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https://doi.org/10.1016/j.ymssp.2018.03.032
0888-3270/

Nomenclature

Roman symbols

a	length in global x-direction, mm
b	length in global y-direction, mm
b'	substituted coefficient, –
f	(natural) frequency, Hz
h	thickness of the rectangular plate, length in global z-direction, mm
h_F^2	driving point admittance, $\mathrm{m}^2/(\mathrm{Ns})^2$
i	imaginary unit, –
m_A	additional mass, kg
t	thickness of the generic car undercarriage, mm
v_0	level reference of the vibration velocity, m/s
$\overline{\tilde{v}^2}$	mean squared rms vibration velocity, $(\mathrm{m/s})^2$
$\underline{\tilde{v}}$	complex rms vibration velocity, m/s
x, y, z	spatial coordinates, m
E	Young's modulus, $\mathrm{N/m}^2$
F	excitation force, N
F_0	level reference of the force, N
L_{Sh}	level of the mean squared transfer admittance, dB
ΔL_{Sh}	deviation of the level of the mean squared transfer admittance, dB
N	number of parameters, –
S	area of the vibrating surface, m^2
Sh_T^2	mean squared transfer admittance, $\mathrm{m}^4/(\mathrm{Ns})^2$
$Sh_{T,0}^2$	level reference of the mean squared transfer admittance, $\mathrm{m}^4/(\mathrm{Ns})^2$
X	design parameter, –
X'	substituted design parameter, –
Y	response function, –
Y'	substituted response function, –

Greek symbols

α	power, –
ε_f	deviation criterion of the natural frequencies, %
η	material loss factor, –
μ	Poisson's ratio, –
ρ	mass density, $\mathrm{kg/m}^3$
ϕ_{X_j}	scaling factor of design parameter X_j, –

Subscripts and superscripts

F	subscript indicating the excitation point
j	index of design parameters
k	index of response functions
l	index of length and width
m	number of half-waves in x-direction
n	number of half-waves in y-direction, mode number of the generic car undercarriage
R	subscript indicating the receiver point
(p)	superscript of a parent
(r)	superscript of a replica
(+)	design parameter that is scaled up during the sensitivity analysis
($-$)	design parameter that is scaled down during the sensitivity analysis

Abbreviations

FE	finite elements
GSA	global sensitivity analysis
LSA	local sensitivity analysis
MSTA	mean squared transfer admittance
MD	model deviation criterion
ODS	operation deflection shape
rms	root mean square
SA	sensitivity analysis
SLD	scaling law deviation criterion