Reliability Engineering and System Safety 158 (2017) 50–57

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

A hybrid approach for global sensitivity analysis

Souvik Chakraborty*, Rajib Chowdhury

Department of Civil Engineering, IIT Roorkee, Roorkee, India

ARTICLE INFO

Keywords:
Global sensitivity analysis
PCFE
Uncertainty analysis
Structural mechanics

ABSTRACT

Distribution based sensitivity analysis (DSA) computes sensitivity of the input random variables with respect to the change in distribution of output response. Although DSA is widely appreciated as the best tool for sensitivity analysis, the computational issue associated with this method prohibits its use for complex structures involving costly finite element analysis. For addressing this issue, this paper presents a method that couples polynomial correlated function expansion (PCFE) with DSA. PCFE is a fully equivalent operational model which integrates the concepts of analysis of variance decomposition, extended bases and homotopy algorithm. By integrating PCFE into DSA, it is possible to considerably alleviate the computational burden. Three examples are presented to demonstrate the performance of the proposed approach for sensitivity analysis. For all the problems, proposed approach yields excellent results with significantly reduced computational effort. The results obtained, to some extent, indicate that proposed approach can be utilized for sensitivity analysis of large scale structures.

1. Introduction

All physical systems have inherent associated randomness. This randomness may exists either in the model formulation [1–4] or in model parameters (aleatoric uncertainty) [1,2]. Naturally these uncertainties propagate and is reflected in the model results and predictions [5]. The knowledge regarding the influence of the input uncertainties on the model response is extremely important to a design engineer. In this regard, the sensitivity analysis is an important tool which quantifies the relative importance of the input variables. Given limited computational resource, sensitivity analysis may be used to identify and eliminate the lesser important variables. Due to this reason, sensitivity analysis has found wide application in stochastic computations [6–10], environmental science [11], geographic information systems [12], physiochemical systems [13,14] etc.

The last two decades have witnessed significant progress in sensitivity analysis. The most widely used sensitivity analysis tools are based on the differential methods [15,16], where the sensitivity analysis is performed by differentiating the output variable with respect to the inputs. The differentiation is generally performed by employing the finite difference method. Although easy to implement, this method is computationally expensive because determining stochastic response corresponding to each system parameter is demanding, specifically for large scale systems that involve costly finite element (FE) analysis. To overcome this issue, researchers have tried to formulate sensitivity index by utilizing direct differentiation [17]. However, it was observed that direct differentiation yields accurate result only when (a) the underlying function is either linear and (b) the design point [18] can be accurately traced. Other possible alternatives include score function based approach [19,20] and perturbation method [21–23]. All these methods come under the broad category of local sensitivity analysis.

A possible alternative to the local sensitivity analysis are the methods based on global sensitivity analysis (GSA) [24–28]. GSA is better suited for stochastic systems because, unlike local sensitivity analysis, GSA does not compute the sensitivity indexes based on only a few selected points. One of the popular approaches for GSA is the Morris method [29]. The basic idea is to vary only one input while keeping the other inputs constant. GSA is computed based on the local variations at different points. However for systems involving uncertainties, only partial information is obtained by employing the Morris method. Another approach for GSA is the variance based sensitivity analysis proposed by Sobol and his associates [30,31]. As the name suggests, this approach utilizes the second moment properties to compute the sensitivity index of a variable. This method is quite popular because of its simplicity.

Another group of methods for studying the sensitivity of a model inputs are the distribution based sensitivity analysis (DSA) tools [11,32–34]. In this method, the sensitivity index depends on the distribution (either probability density function or cumulative distribution function) of the response. It is a well-known fact that sensitivity index computed using DSA is more appropriate as it reflects the overall influence of the input variables on the output response. However, computational inefficiency of this method has prohibited its use in mechanics oriented problems.

Motivated by the limitations highlighted above, present study focusses on developing an efficient algorithm for distribution based GSA. To be specific, proposed approach couples polynomial correlated function expan-

* Corresponding author.
E-mail address: csouvik41@gmail.com (S. Chakraborty).

http://dx.doi.org/10.1016/j.ress.2016.10.013
Received 30 October 2015; Received in revised form 3 September 2016; Accepted 22 October 2016
Available online 25 October 2016

sion (PCFE) [35–41] with DSA. This coupling results in a significant reduction of the computational cost. Furthermore unlike most methods, proposed method is capable of performing sensitivity analysis of system entailing both dependent and independent random variables without the need of any *ad hoc* transformations.

The rest of the paper is organised as follows. In Section 2, the GSA has been reviewed. Section 3 describes the basic formulation of PCFE. In Section 4, an algorithm for integrating PCFE into the framework of DSA has been proposed. Computational efficiency of the proposed approach has also been explained. Application of the proposed approach for sensitivity analysis has been demonstrated in Section 5. Finally, Section 6 provides the concluding remarks.

2. Global sensitivity analysis: a review

In this section, a brief description of global sensitivity analysis has been provided. Suppose, the uncertain input variables of a system are represented by an N dimensional vector $\mathbf{x} = \{x_1, x_2, \ldots, x_N\} \in \mathbb{R}^N$. Given the uncertainty in $\mathbf{x}$, it is obvious that the output response Y would also be uncertain with the variability of Y consisting of different components of $\mathbf{x}$. The objective of present study is to evaluate these contributions, such that one have a clear idea about the relative importance of the input variables. Saltelli and Tarantola [42] conferred that in sensitivity analysis based on variance "We are asked to bet on the factor that, if determined (*i.e.,* fixed to its true value), would lead to the greatest reduction in the variance of Y". Hence,

$$\mathcal{S}_i = \frac{Var(Y) - \mathbb{E}(Var(Y|x_i))_{x_i}}{Var(Y)} \tag{1}$$

where $\mathbb{E}(\bullet)$ and $Var(\bullet)$, respectively, denotes expectation and variance operators. $\mathcal{S}_i$ in Eq. (1) denotes the sensitivity index of the ith variable.

On contrary in DSA, "We are asked to bet on the factor that, if determined would lead to the greatest expected modification in the distribution of Y" [43]. Distribution based sensitivity measure was first proposed by Park and Ahn [44]. The sensitivity measure considered in this study takes the following form [11]:

$$\delta_i = \frac{1}{2}\mathbb{E}\left(\int_{\Omega_Y} |\varpi(Y) - \varpi(Y|x_i)| dy\right)_{x_i} \tag{2}$$

where Ω_Y is the domain of the output response Y and $\varpi(\bullet)$ denotes the probability density function. A visual representation of the distribution based sensitivity measure is shown in Fig. 1.

Remark 1. As pointed out by Borgonovo [11,33], the sensitivity index described in Eq. (2) has two desirable properties. Firstly, δ_i is individually and jointly normalised, *i.e.,* $0 \le \delta_i \le 1$ and $\delta_{12\ldots N} = 1$. Secondly, δ_i is invariant to monotonic transformation.

Remark 2. If input variable x_i and response Y are independent, $\delta_i = 0$ [11,33].

Remark 3. Iman and Hora [45] pointed out that in presence of long input/output, the statistical quantifies (such as variance) obtained might not be robust. As a consequence, variance based sensitivity measures loses its robustness. Distribution based sensitivityindices are free from this problem.

Remark 4. Computation of sensitivity index by Eq. (2) involves extremely large number of actual function evaluations. Due to this reason, use of this method is limited to models where thousands of model evaluations are possible within a feasible computer time.

One possible alternative for addressing the above mentioned issue is the application of surrogate model [11,46,47] to replace the original model. In this work, an efficient fully equivalent operational model, referred here as PCFE, has been used to replace the original model. The detailed description of this method is provided in next section.

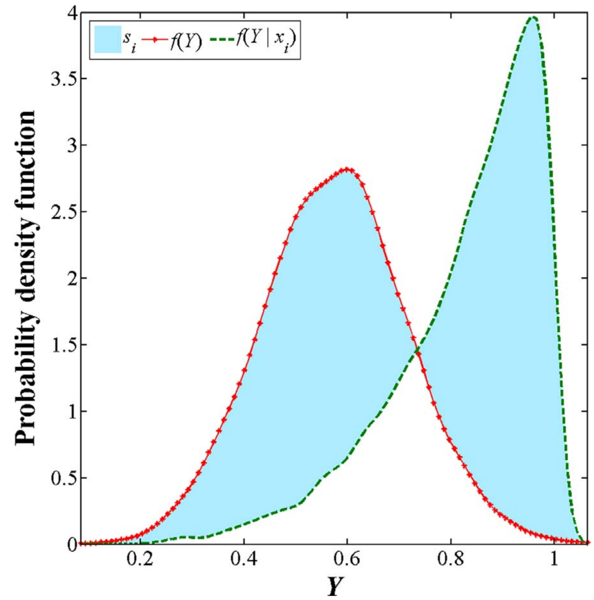

Fig. 1. Visual representation of DSA. s_i is the difference between the unconditional PDF $f(Y)$ and conditional PDF$f(Y|x_i)$, obtained by fixing the variable x_i.

3. Polynomial correlated function expansion

Polynomial correlated function expansion (PCFE) [35–41] is a fully equivalent operational model recently developed for capturing the high dimensional relationship between sets of input and output model variables. It can be viewed as an extension of classical functional ANOVA decomposition [48] where the component functions are represented by utilizing the extended bases [49]. In literature, PCFE is also referred as generalised high dimensional model representation [50]. The unknown coefficients associated with the bases are determined by employing a homotopy algorithm (HA) [51–53]. HA determines the unknown coefficients by minimizing the least squared error and an objective function. The objective function defines an additional criteria that is enforced on the solution. In PCFE, the hierarchical orthogonality of the component functions is considered to be the additional criteria.

Let, $\mathbf{x} = \{x_1, x_2, \ldots. x_N\}$ be a N dimensional vector, representing the input variables of a structural system. It is quite logical to express the output Y as a finite series as [31]

$$Y(=g(\mathbf{x})) = g_0 + \sum_{k=1}^{N} \sum_{i_1<i_2\cdots<i_k} g_{i_1 i_2 \ldots i_N}(x_{i_1}, x_{i_2}, \ldots x_{i_N}) = g_0 + \sum_{i_1=1}^{N} g_i(x_i) + \sum_{1\le i_1<i_2\le N} g_{ij}(x_i, x_j) + \cdots + g_{12\ldots N}(x_1, x_2, \ldots, x_N) \tag{3}$$

where g_0 is a constant and termed as zeroth order component function.

Definition 1. : Assume, two subspace R and B in Hilbert space are spanned by basis $\{r_1, r_2, \ldots, r_1\}$ and $\{b_1, b_2, \ldots, b_m\}$ respectively. Now if (i) $B \supset R$ and (ii) $B = R + R^{\perp}$ where, $R^{\perp}$ is the orthogonal complement subspace of R in B, we term B as extended basis and R as non-extended basis [50].

Now if ψ be some suitable basis for $\mathbf{x} \subseteq \mathbf{X}$, where $\mathbf{X}: = \{1, 2, \ldots, N\}$, Eq. (3) can be expressed as [35,49]:

$$g(\mathbf{x}) = g_0 + \sum_{k=1}^{N}\left\{\sum_{i_1=1}^{N-k+1} \cdots \sum_{i_k=i_{k-1}}^{N} \sum_{r=1}^{k}\left[\sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty}\cdots\sum_{m_r=1}^{\infty} \alpha_{m_1 m_2 \ldots m_r}^{(i_1 i_2 \ldots i_k) i_r} \psi_{m_1}^{i_1} \cdots \psi_{m_r}^{i_r}\right]\right\} \tag{4}$$

where α indicates the unknown expansion coefficients. However, Eq. (4) represents an infinite series and needs to be truncated. Considering upto