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Dynamics of structures with uncertain-but-bounded parameters via pseudo-static sensitivity analysis

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ABSTRACT

This paper deals with the analysis of linear-elastic structures with uncertain-but-bounded parameters subjected to deterministic dynamic loads. A novel procedure based on the use of sensitivity analysis in conjunction with classical modal superposition method is proposed. Specifically, a *pseudo-static sensitivity* analysis is performed to seek the combinations of the endpoints of the uncertain parameters which give the *lower bound* and *upper bound* of the response at each time instant. Among these, two combinations are selected as the most common ones over the time interval of interest in order to avoid the onerous updating of the uncertain parameters at each time instant. Then, the bounds of the response time-history are evaluated by performing two parallel deterministic modal analyses associated to the most common combinations of the extreme values of interval uncertainties.

Numerical results demonstrate that the proposed method is more efficient than both the *Interval Perturbation Method* (*IPM*) and the classical combinatorial procedure. Furthermore, unlike the *IPM*, it allows the analysis of large-size structures exhibiting relatively large fluctuations of the uncertain parameters.

1. Introduction

It is well-known that almost any type of structural system is subjected to dynamic loads during its lifetime. Typical dangerous dynamic loads may be of impulsive nature (blast or explosion) or have long-duration, such as those resulting from earthquakes or wind. A structural dynamic problem differs to a large extent from its static loading counterpart. The main difference lies in the time-varying nature of both loading and response. It follows that a dynamic problem does not admit a single solution, but a sequence of solutions at each time instant of the response history. In the context of static analysis, structural response is evaluated as solution of algebraic equations, whereas a dynamic problem is governed by *Ordinary Differential Equations* (*ODEs*). Thus, a dynamic analysis is clearly more complex and time-consuming than a static one [1].

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The main purpose of structural analysis is to evaluate response quantities, such as displacements, rotations, stresses etc., to perform reliability assessment of engineering systems. To this aim, not only external loads but also material properties need to be modeled with great accuracy. It has been widely recognized that parameters characterizing the mechanical behavior of structural materials are affected by uncertainties caused by measurement or manufacturing errors, or other factors. Therefore, in order to accurately predict structural performance, the non-deterministic nature of input parameters has to be taken into account by using suitable mathematical models. Experimental data available to define mechanical properties are often quite limited, so that the accuracy of the traditional probabilistic model of uncertainty becomes questionable. Indeed, within a probabilistic framework, the uncertain parameters are modeled as random variables or random fields, under the assumption that complete information is available to define the pertinent probability density function (PDF). However, as highlighted by Ben-Haim and Elishakoff [2], even small changes in the PDF of the input parameters are prone to produce large changes in the failure probability.

Non-probabilistic approaches can be alternatively used to treat uncertainties affecting material properties when only fragmentary or incomplete experimental data are available [3,4]. In this framework, the interval model seems today the most suitable analytical tool when only range information or tolerance is known [5,6]. The interval model, which is based on the set theory, represents the uncertain parameters as interval variables with assigned *lower bound* (*LB*) and *upper bound* (*UB*) without requiring complete information on the distribution of the uncertainties between such bounds.

The main advantage of the *Classical Interval Analysis* (*CIA*) is that it provides analytically rigorous enclosures of the solution [5,6], but its application to practical engineering problems is not an easy task due to two main drawbacks commonly faced in the development of interval-based procedures for structural analysis: *i*) the drastic overestimation of the interval solution range due to the so-called *dependency phenomenon* [3,6]; *ii*) the high computational costs required by uncertainty propagation procedures, such as the classical combinatorial approach [3,7].

The dynamic equilibrium of structural systems with interval uncertainties is governed by sets of interval *ODE*s. If such equations are solved using the *CIA*, the overestimation will progressively increase in the process of numerical iterations [8,9]. Several methods have been proposed in literature to limit the overestimation of the solution of the set of *ODE*s governing linear or non-linear dynamic problems with interval uncertainties. In particular, the upper and lower bounds of the dynamic response were obtained by Chen et al. [10] using Taylor series expansion combined with a method based on matrix perturbation theory. Subsequently, the *Interval Perturbation Method* (*IPM*), which relies on Taylor series expansion and parameter perturbation, has been introduced to evaluate the interval dynamic response of structures subjected to deterministic [11–13] or stochastic excitations [14]. More recently, Gao et al. [15] presented the *interval factor method* to calculate the dynamic response of truss structures with interval parameters under interval loads. Rama Rao et al. [16] proposed two methods to obtain the transient response time-history of structures subjected to a sudden impact load: the first one is based on adaptive Taylor series expansion along with gradient method, while the second approach relies on direct optimization. By finite element method and mathematical programming theory, Qiu and Ni [17] developed a new inequality model for determining the interval dynamic response. Liu et al. [18] investigated the dynamic response of vehicle-bridge interaction system with bounded parameters by using the interval analysis method and an improved particle swarm optimization algorithm. The task of determining extreme dynamic responses of complicated vehicle-bridge interaction systems with interval uncertainties was also addressed by Zou et al. [19] who used first-order Taylor series expansion and particle swarm optimization algorithm. The subinterval technique was used to improve the accuracy of the predicted bounds of bridge deflection and bending moment. Yang et al. [20] used the Laplace transform to convert the *ODE*s into a system of linear equations and the inverse Laplace transform to obtain the response time-history, once higher-order terms are removed by matrix perturbation technique in Laplace domain. Wu et al. [9] introduced Chebyshev series expansions into the interval framework to develop a new method for the analysis of non-linear dynamic systems able to handle degrees of uncertainty larger than those allowed by first-order interval Taylor series expansion. The Chebyshev interval method was also used by Wei et al. [21] to investigate the dynamic responses of a geared transmission system with uncertain parameters. Xia et al. [22] proposed a Monte Carlo method based on the Chebyshev polynomial expansion to predict the range of the dynamic response of structures with uncertainties modeled as time-variant interval processes. Wang et al. [23] presented a new non-probabilistic time-dependent reliability method for vibration active control systems with uncertain-but-bounded parameters. More recently, inverse methods for dynamic loads identifications were developed by interval mathematics [24,25].

Though often less accurate than other methods, the *IPM* or, equivalently, the first-order interval Taylor series expansion, is the most widely used procedure to obtain the *LB* and *UB* of the interval dynamic response. The main advantages of the *IPM* are the flexibility and the simplicity of the mathematical formulation. However, since the effect of neglecting higher-order terms is unpredictable, the effectiveness of this method is limited to uncertainties with small intervals. Furthermore, the computational burden associated to the *IPM* rapidly increases with the dimensions of structural systems and the number of uncertainties.

In this paper, a novel procedure to evaluate the bounds of the interval response of structural systems with uncertain-but-bounded material properties under deterministic dynamic excitations is presented. The key idea of the proposed approach is to properly extend sensitivity-based procedures available in literature to perform interval static structural analysis [26,27] so as to address interval dynamic problems. Indeed, at each time instant, the dynamic response of a structural system is a monotonic function of the uncertain material properties. In this context, two main issues need to be faced: the high computational burden required by the evaluation of response sensitivities to the uncertain parameters and their time-dependency. To overcome such limitations, the present study introduces the definition of *pseudo-static sensitivity* of the response which