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Load capacity probabilistic sensitivity analysis of thin-walled beams

Maria Kotełko[a,*], Paweł Lis[a], Martin Macdonald[b]

[a] Łódź University of Technology, Department of Strength of Materials, Łódź, Poland
[b] Glasgow Caledonian University, School of Engineering & Built Environment, Glasgow, Scotland, UK

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ABSTRACT

The paper deals with the load carrying capacity probabilistic variance-based sensitivity analysis of thin-walled box-section girders subjected to pure bending. The lower- and upper-bound load capacity estimation is performed using two different analytical methods. The sensitivity analysis performed is based on the methodology of the Monte-Carlo method. The analysis is carried out using the polynomial decomposition and multi-dimensional linear regression. The sample results obtained are presented in diagrams and pie charts showing the sensitivity of load capacity to different random input variables (material properties and geometrical parameters). The variance-based analysis (Anova) of lower-bound and upper-bound load capacity estimation is carried out, from which some conclusions are derived, if (and how) assumed changes in standard deviations of input variables influence the magnitude of the load capacity and differences in upper-bound and lower-bound load capacity estimations. The results of Anova tests are shown in sample histograms. Some final conclusions concerning the efficiency of the applied models and the statistically significant influence of input random variables (yield stress, wall thickness, height and length of the beam) upon the upper-bound and lower bound estimation of the load capacity as well as the difference of these two estimations, are presented.

1. Introduction

The load carrying capacity of thin-walled structures (TWS) is influenced by a number of material and geometrical parameters, which can be of random character. Thus, the application of sensitivity and uncertainty analysis is of use in the evaluation of the influence of variability of different geometrical and material imperfections on the variability of the load carrying capacity of TWS.

The random character of material parameters comes not only from the manufacturing process of metal sheets, but in the case of cold formed thin walled members, from the forming process (e.g. cold rolling). Material properties are sometimes substantially different from those of the steel, strip, plate or bar before forming. The reason for this is that the cold forming operation increases the yield and tensile strengths, and at the same time decreases the ductility (Kankanamge and Mahendran 2011 [1]; Macdonald, Rhodes and Taylor 2000 [2]; Chen and Young 2004 [3]). This behaviour is highlighted in Fig. 1.

The mechanical properties are also different in various parts of the cross-section. This is due to the increase in the yield strength and the tensile strength of the material in the corners of the section which are considerably higher than the material in the flat elements (Karren and Winter [4]). Reasons for geometrical imperfections may be different: manufacturing process of TWS members, welding process, etc.

Sensitivity analysis methods are of two types: deterministic and stochastic (probabilistic). The deterministic sensitivity analysis, which may be referred to as a parametric study, is widely used in structural design. However, using this approach does not always obtain an answer about the sensitivity or uncertainty of output parameters in terms of input data. Using the probabilistic sensitivity analysis, the input parameters that have a predominant influence on the uncertainty of the output variables (e.g. load capacity) can be determined.

In recent years the deterministic approach to the design of TWS has often been replaced by the probabilistic one. It particularly concerns thin-walled girders. Also, some new design codes, particularly concerning TWS in civil engineering, treat the structural reliability and load carrying capacity of TWS as a probabilistic problem. However, in using any probabilistic method, a great number of calculations have to be performed, and the main limitation becomes the time of computation, which depends on the method applied.

As far as an applied calculation apparatus is concerned, methods of probabilistic sensitivity analysis can be divided into two groups: analytical or analytical-numerical methods and "purely" numerical methods, mainly the Finite Element Method (FEM) [47].

The strength of thin-walled structures is usually calculated on the basis of the "effective width" model and the ultimate capacity is evaluated using a reduced or effective cross-section and, additionally,

* Corresponding author.
E-mail address: maria.kotelko@p.lodz.pl (M. Kotełko).

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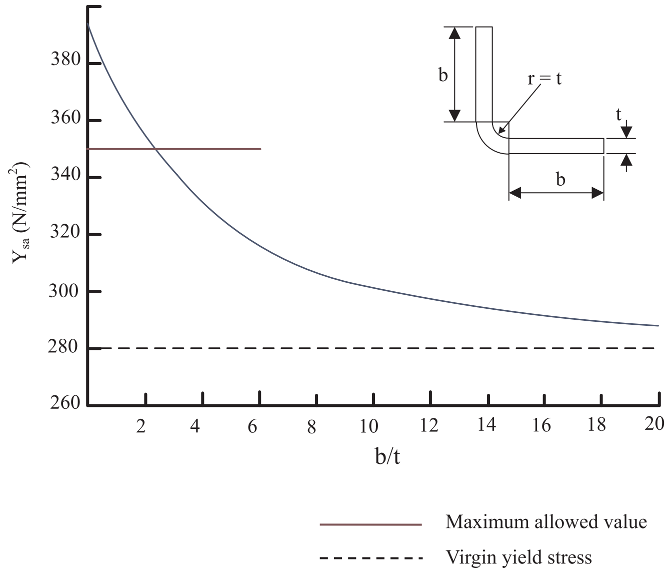

Fig. 1. Variation in predicted average yield strength for a cold-formed bend with variation in geometry ([2]).

the elastic limit for maximum stress. This approach is currently used in almost all design codes and leads to the lower-bound estimation of the load-carrying capacity. The elastic post-buckling behaviour of a thin-walled beam was analysed by Kolakowski et al. [5], who solved the problem using the asymptotic method in the range of the second order approximation. The algorithm based on the asymptotic method is relatively simple and delivers the lower-bound estimation of the load-carrying capacity in a shorter computation time.

However, TWS members display a significant post-elastic capacity. This means that the actual load carrying capacity of any thin-walled member is higher than the ultimate load calculated using the method mentioned above.

Thus, the alternative approach is the upper-bound estimation of the load carrying capacity, consisting of the determination of the intersection point of a post-buckling path, evaluated using either analytical or numerical method, and a rigid-plastic *failure curve* obtained from the plastic mechanism analysis (Kotełko et al. [6,7]).

The present paper deals with the sensitivity analysis of the load carrying capacity of a thin-walled, box-section beam subjected to pure bending as shown in Fig. 2. The sensitivity analysis is performed both for lower-bound and upper-bound estimation of the load carrying capacity. Two analytical-numerical methods are applied to calculate the lower-bound and upper-bound load carrying capacity of the beam.

2. State of the art of the subject

First attempts to apply a probabilistic approach to sensitivity analysis go back to the 1970s. One of the pioneering papers was published by Fukumoto et al. in 1976 [8], as well as Arorra (1992 [9,10]). In Poland, the pioneers in this area were Mróz (1983 [11,12]), Szymczak (1991 [13,14]) and Szefer (1983 [15]). Mróz [11] and Szefer [15] applied the probabilistic approach to the sensitivity analysis of non-linear mechanical systems.

In the 1980s, the first results of research into the application of the probabilistic approach to the analysis of stability of structures were published. In Poland, Murzewski [16] analysed the problem of random instability of high structures (building structures and cranes). The random variables were the dead weight of the structures and wind force. He also applied the probabilistic approach to the problem of conjunction of two independent random events, i.e. elastic buckling and plastic failure of axially compressed columns [17]. The same approach was continued in other research work concerning elastic-plastic buckling of frames with imperfections (Murzewski [18]). In [13,14] the sensitivity analysis of load-capacity of I- beams subjected to torsion was analysed, while in [20] a similar problem concerning beams of bi-symmetrical cross-section was considered. A detailed discussion of problems related to sensitivity analysis of beams and frames was published in [1] and by Szymczak in [19], as well as in [21,22].

The probabilistic sensitivity analysis in the domain of thin-walled structures was up until now applied to the estimation of load capacity of beam-columns of I–section mainly. Some analyses have been performed for columns in compression.

Z. and J. Kala [25] applied the probabilistic sensitivity analysis to the buckling and limit load analysis of I-section columns under compression, taking into account the variance change of initial geometric imperfection. The analysis is based on the Sobol method. Puklicky and Kala [23] performed the sensitivity analysis into limit states and load capacity of composite I-section columns (filled with concrete) used in building engineering. Long columns and global buckling of columns was considered. FE simulations and LHS method (**L**atin **H**ypercube **D**istribution) were applied. The random variables were both material parameters and geometrical imperfections. They stated that the results of the sensitivity analysis, which took into account the load capacity redundancy in the post-elastic state, can be used in load-carrying capacity calculation, including standard recommendations.

The fundamental works in the domain of the sensitivity analysis of beams were published by Z.Kala, J.Kala, Skaloud and Melcher [23–37].

Szymczak [19] performed first-order sensitivity analysis based on the variational approach. The subjects of this analysis were thin-walled beams of bi-symmetric cross-section, subjected to combined load (bending, compression and torsion). Analysis has been extended onto thin-walled frames under torsion. In [21], Szymczak et al. presented the review of sensitivity analysis solutions based on the variational approach, concerning thin-walled beams of mono- and bi-symmetrical cross-sections, subjected to combined loading. The output variables were deflections of beams, sectional forces, as well as modes and frequencies of torsional vibrations.

Kala, Skaloud et al. [26] carried out a sensitivity analysis of I-section beam load capacity in the pre-buckling state were they used the FE method. The random variables were material parameters and wall thickness. They did not state any influence of input random variables on the load capacity itself (whether it increases or decreases) and they did

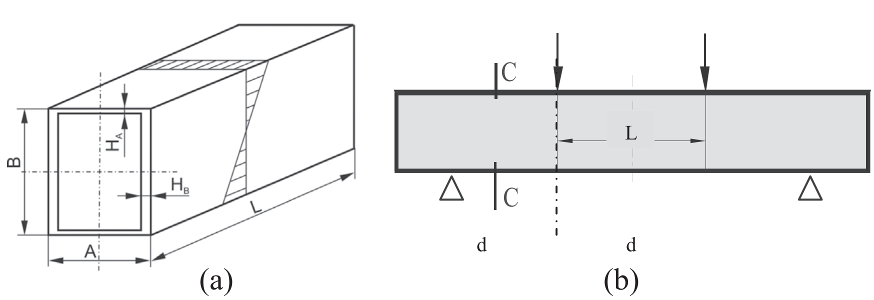

Fig. 2. Box-section girder under pure bending: (a) dimensions; (b) load and support lay-out.