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Momentum in dynamic contests☆

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ABSTRACT

We consider a two-player advertising race subject to momentum. Momentum is modeled as a complementarity between current and past campaign spending in a way that is reminiscent of models of addiction and habit formation: the more effective a player's past spending has been, the more effective her future spending will be. For symmetric races in which the effectiveness of spending is of the Cobb-Douglas form, we derive analytic solutions for the equilibrium path. Our theory rationalizes alternative campaign strategies including aggressive openings and the development of a warchest for a final campaign assault.

1. Introduction

We examine the optimal sequencing of a scarce advertising budget in contests subject to momentum. Our premise is that the effectiveness of advertising improves with the receptiveness of the audience. For example, consumers pay more attention to products that interest them; and voters are more likely to be swayed by the arguments of candidates they prefer. In the presence of such momentum, should combatants commence early and devote substantial resources to kickstart the advertising campaign, or would they be better served by saving resources for a vigorous assault at the end of the campaign?

For clarity, we maintain a running example of a political contest throughout the paper, although one should keep in mind that our approach lends itself to other applications. Related tradeoffs arise in the marketing of competing products, such as the looming launch of two rival game consoles or the release of two potential blockbuster movies in peak season. Sporting contests also give rise to similar tensions. An early lead by one team may provide a psychological boost to the leading team. Should finite energy and the most talented players be mobilized at kickoff or should they be spared for the closing stages of the contest? We provide an equilibrium theory of the dynamic sequencing of finite resources in such contests.

Political campaign strategy is a topic of frequent debate and a great deal of money.[1] The dynamic sequencing of resources is an important element of campaign strategy that has received little formal attention. We frame the campaign spending decisions of a political candidate in a manner analogous to the advertising decisions of a firm.[2] In our setting, each candidate decides how to sequence a limited campaign budget over time in order to maximise her electoral chances. The shape of the dynamic spending profile is influenced by three principal considerations: decay, diminishing returns, and momentum. Decay reflects the increased electoral salience of campaign events that occur closer to election day. Recent campaign events are likely to be fresh in the minds of voters on polling day, encouraging combatants to nurture a war chest for a final campaign push. The presence of diminishing returns restricts the capacity of the electorate to receive and absorb messages, placing a limitation on the effectiveness of intense campaigning. Careful, targetted campaigning may be able to reach the most receptive and available of voters at minimal cost. However, even saturation coverage may not be able to influence or even attract the attention of some voters. This provides an incentive to smooth campaign spending over time.

Decay and diminishing returns are common features in advertising models, while momentum is a novel inclusion. To illustrate the role of momentum, consider a race between two politicians running for the same office. Suppose that one of them has succeeded in getting ahead

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[1] As Denter and Sisak (2015) note, more than USD 2 billion was spent on the 2012 United States Presidential Campaign.

[2] The correspondence between advertising and campaigning is not a novel insight. See, for example, Snyder (1989).

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in the polls, thus having gained momentum over her opponent. Then, one might expect that it will be easier, loosely speaking, for the politician leading the polls to maintain her lead and, perhaps, reinforce it. For example, if from this point onwards until election time both politicians match each other's campaign efforts, we might expect the leading politician to not only maintain her lead to the end, but strengthen it even further. In our model, momentum operates by increasing the effectiveness of campaigning for popular candidates. This provides a payoff to an aggressive early campaign strategy.[3]

Momentum effects may arise due to different underlying forces and our formulation does not take a position on the precise source. In a political campaign, momentum may arise due to the electorate's imperfect memory. More specifically, Mullainathan (2002) points to a large body of psychological research suggesting that memory operates on the principles of *cue-dependence* and *rehearsal*. According to cue-dependence, events tend to trigger memories of similar past events. As these past events are triggered, they get rehearsed and thus their memories are strengthened. In turn, this makes it even more likely that they will be further rehearsed by similar events that may follow in the future. One can thus interpret a lead by one politician in the polls as that politician having created a superior stock of good memories in the electorate's mind, relative to her opponent. As a result, due to the principles of cue-dependence and rehearsal, her future advertising spending will be, all else equal, more successful than her opponent's, as it will be rehearsing a superior stock of existing memories; i.e. the leading politician has gained momentum.

Alternative forces may also give rise to momentum. For instance, in the context of our console example, momentum may arise if consumer interest is piqued by prior advertising, encouraging consumers to pay more attention to subsequent advertising. Network effects could also play a role. Consumers may pay more attention to a console that is showing early signs of popularity. In a political contest, a voter who forms a favorable impression of a candidate is more likely to engage in favorable discussions about the candidate with other voters. Taking this argument further, we could interpret the establishment of a successful grass-roots campaign as a contribution to momentum. Indeed, political candidates may target campaign efforts towards constituents likely to have a greater impact on momentum. In the context of technology or sporting contests, the leader may be able to demoralise her opponent, reducing the effectiveness of her efforts.

Formally, we incorporate momentum by imposing the following assumption: the more effective a politician's past spending has been, the more effective her current and future spending will be. To use the economics jargon, there exists a complementarity between current and past campaign spending. In this sense, we model the notion of momentum in a way that is reminiscent of addiction and habit formation models, such as Becker and Murphy (1988) and Constantinides (1990), respectively.

Beyond the immediate application to political contests, our model provides a vehicle for understanding the role of momentum in games. Our baseline symmetric Cobb-Douglas formulation gives rise to a closed form solution for the equilibrium advertising path. Depending on the relative intensity of momentum and decay, the equilibrium path can be increasing, decreasing, or U-shaped. Our model also provides a useful framework with which to study more general games incorporating momentum. We illustrate by considering numerical solutions to an asymmetric version of the model.

The starting point of our theory is that campaign spending is effective. In the context of political campaigns, this position is not without controversy (see, for example, Holbrook (1996), Shaw (1999), and Jacobsen (2015) for a discussion).[4] Erikson and Palfrey (2000) argue that the inherent difficulty of empirically inferring the causal relationship between campaign spending and outcomes may be to blame: if campaign success itself influences campaign spending, it is challenging to isolate the effectiveness of campaign spending. Our model provides an illustration. Despite our presumption of effective campaigning, equilibrium behaviour may not induce a correlation between campaign intensity and campaign outcomes.

1.1. The set up and main results

To set the stage, consider a single politician with a fixed advertising budget. Following Nerlove and Arrow (1962), we allow advertising to generate a stock of goodwill, which decays at a fixed rate, δ. Our politician wishes to maximise her stock of goodwill at a critical time, T (e.g. at election time). The marginal effect of her advertising spending in each period depends on her existing stock of goodwill. That is, it is possible for her to create momentum during the campaign. How should she then sequence her scarce advertising budget to maximise her stock of goodwill at election time?

We show that this problem is isomorphic to a standard inter-temporal utility maximization model with addiction or habit formation and there is nothing here we claim to be novel. We thus view the single-agent problem as a stepping stone towards the two-agent game. As we will see, it also provides a useful benchmark. We show that under a payoff function of the Cobb-Douglas form (with the two arguments being the advertising flow and the existing stock of goodwill), the politician's optimal strategy is to advertise at a (weakly) increasing exponential rate as in the model with no momentum, with this rate being (weakly) decreasing in the parameter that measures the magnitude of the momentum effect.[5] In other words, relative to the model with no momentum effects, the optimal path has the same qualitative features, but is *flatter*. With an increasing path of spending, a flatter path is equivalent to a more aggressive initial campaign: the politician strives to build early on a healthy stock of goodwill that will enhance the impact of future campaigning.

We then analyze a differential game between two such politicians. Each politician now nurtures a separate stock of goodwill. Not only must each politician consider the effect of her stock of goodwill on her own advertising effectiveness, but she should also contemplate the impact her advertising has on her rival's spending pattern and stock of goodwill, as well as the manner in which this feeds back into her own stock of goodwill.

We assume that each politician's objective is to maximize the lead over her opponent at election time. An alternative formulation that might be of some interest is a binary objective function. This would be the case if the politician simply wanted to win the election by being ahead on election day. We think our formulation is of interest for at least two reasons (but see Snyder (1989) for an example where these goals can lead to different outcomes). First, our politician will care, at least to some extent, about the margin of her victory as this will influence her mandate for action (and potentially stoke her ego). Indeed, in proportional representation settings, the margin of victory has obvious importance. Second, even if our politician is primarily concerned about winning per se, in a stochastic setting she will be more secure in her likelihood of winning the greater her expected margin over her opponent. Similar considerations apply in marketing and sporting settings.

[3] While campaigns typically set aside funds for the closing stages of a race, Lizza (2012) notes that an early aggressive advertising campaign has been a feature of many Presidential races over the last 40 years.

[4] We do not, however, take a position on the source of campaign effectiveness. By contrast, Box-Steffensmeier et al. (2009) emphasise the importance of careful account of the co-dependence of principal actors in a campaign including political candidates, the media, and voters. While we do not explicitly model these relationships, our model presumes a feedback relationship between campaign spending and voter support.

[5] The "weakly" qualifier is needed because the optimal strategy is to advertise at a constant rate, regardless of the magnitude of the momentum effect, if and only if the stock of goodwill does not decay (i.e. $\delta = 0$).