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Robust minimum variance portfolio optimization modelling under scenario uncertainty[☆]

Panos Xidonas[a,*], Christis Hassapis[b], John Soulis[c], Aristeidis Samitas[d]

[a] ESSCA Grande École, France
[b] University of Cyprus, Cyprus
[c] Imperial College London, United Kingdom
[d] University of the Aegean, Greece

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ABSTRACT

Our purpose in this article is to develop a robust optimization model which minimizes portfolio variance for a finite set of covariance matrices scenarios. The proposed approach aims at the proper selection of portfolios, in a way that for every covariance matrix estimate included in the analysis, the calculated portfolio variance remains as close to the corresponding individual minimum value, as possible. To accomplish this, we formulate a mixed-integer non-linear program with quadratic constraints. With respect to practical underlying concerns, investment policy constraints regarding the portfolio structure are also taken into consideration. The validity of the proposed approach is verified through extensive out-of-sample empirical testing in the EuroStoxx 50, the S & P 100, the S & P 500, as well as a well-diversified investment universe of ETFs. We report consistent generation of stable out-of-sample returns, which are in most cases superior to those of the worst-case scenario. Moreover, we provide strong evidence that the proposed robust model assists in selective asset picking and systematic avoidance of excessive losses.

1. Introduction

In the last 20 years, the research activity in robust portfolio optimization is immense (Ghahtarani and Najafi, 2013; Mansini et al. 2014; Ayub et al. 2015; Gorissen et al. 2015). Kolm et al. (2014) reviewed the 60-year course of portfolio optimization and confirmed the persistent portfolio robustness trend that has emerged. Results of Google Scholar queries provide some interesting figures that highlight this current thriving momentum: When searching for *modern portfolio theory*, we obtained 404,000 results, when searching for *portfolio optimization*, we obtained 241,000 results and when searching for *robust portfolio optimization*, we obtained 48,700 results. Hence, we note that there is a constantly growing underlying research momentum in the field of robust portfolio optimization.

Recent developments in the field of robust portfolio theory imply that the knowledge of future returns and variances, delivered by classic point-estimation techniques, cannot be thoroughly trusted. Since risk and return are characterized by randomness, one should keep in mind that problem data could be described by a set of scenarios. Mulvey et al. (1995) were the first to work on models of mathematical optimization where data values come in sets of scenarios, while explaining the concept of robust solutions and introducing the robust model formulation.

Tütüncü and Koenig (2004) described asset's risk and return using continuous uncertainty sets and developed a robust asset allocation program solved by a saddle-point algorithm. Also, Pinar and Tütüncü (2005) introduced the concept of robust profit opportunity in single-period and multi-period formulations. Likewise, multi-period portfolio optimization formulations with additional transactional constraints are found in Bertsimas and Pachamanova (2008). Other recent critical works in the field of robust portfolio optimization are those of DeMiguel and Nogales (2009), Rustem and Howe (2009) and Qiu et al. (2015).

While robust optimization is intended to protect the portfolio against uncertainty, Gregory et al. (2011) calculated that it comes with costs in terms of return. In terms of risk, Huo et al. (2012) proposed robust covariance measures to be included in the portfolio optimization process, so as to generate covariance estimates stable and insensitive to outliers. In order to deal with output fluctuations and stress testing with respect to uncertainty in input data, a study of robustness of

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[*] Corresponding author.

E-mail address: panos.xidonas@essca.fr (P. Xidonas).

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Table 1
Euro Stoxx 50, results.

Run	Out-of sample period	Naïve	MR	3 m mV	6 m mV	1y mV	Robust mV	t
01	01/01/10–29/01/10	-5,125%	−6,121%	−5,695%	1,210%	−5,082%	−4,029%	0,258
02	05/02/10–26/02/10	−1,396%	5,092%	1,452%	1,021%	0,634%	1,005%	0,276
03	05/03/10–26/03/10	8,217%	12,795%	6,422%	5,120%	4,429%	5,025%	0,306
04	02/04/10–30/04/10	−3,390%	−8,994%	−5,554%	−6,968%	−2,451%	−3,269%	0,250
05	07/05/10–28/05/10	−7,301%	−3,991%	−3,739%	−1,984%	−4,195%	−3,328%	0,194
06	04/06/10–25/06/10	0,736%	3,379%	3,672%	2,918%	0,475%	0,571%	0,245
07	02/07/10–30/07/10	4,504%	−5,269%	−0,548%	−1,346%	0,516%	1,316%	0,174
08	06/08/10–27/08/10	−4,063%	−7,591%	−3,958%	−1,170%	−2,966%	−2,111%	0,122
09	03/09/10–24/09/10	6,442%	5,273%	6,303%	4,219%	1,999%	4,309%	0,334
10	01/10/10–29/10/10	1,969%	6,427%	4,212%	3,305%	4,250%	3,645%	0,126
11	05/11/10–26/11/10	−3,429%	6,439%	−4,005%	−3,191%	−3,738%	−3,679%	0,354
12	03/12/10–31/12/10	2,318%	−0,675%	2,145%	0,829%	1,498%	1,918%	0,343
	Average	−0,043%	0,564%	0,059%	0,330%	−0,386%	0,114%	0,249
	Std	4,66%	6,63%	4,36%	3,31%	3,10%	3,16%	0,077

Table 2
S & P 100, results.

Run	Out-of sample period	Naïve	MR	3 m mV	6 m mV	1y mV	Robust mV	t
01	01/01/10–29/01/10	−1,939%	7,009%	3,643%	−0,870%	−1,539%	0,284%	0,5441
02	05/02/10–26/02/10	1,451%	8,303%	1,356%	1,133%	0,714%	0,898%	1,1968
03	05/03/10–26/03/10	2,998%	18,058%	1,498%	0,108%	0,128%	−0,099%	0,6103
04	02/04/10–30/04/10	0,489%	−6,061%	−0,511%	−1,614%	−0,942%	−1,780%	0,7019
05	07/05/10–28/05/10	−4,618%	−4,861%	−3,305%	−3,206%	−2,497%	−3,514%	0,5847
06	04/06/10–25/06/10	−0,233%	−8,355%	−0,194%	2,190%	1,084%	0,973%	0,5123
07	02/07/10–30/07/10	1,730%	−2,555%	0,815%	0,251%	0,699%	0,906%	0,2614
08	06/08/10–27/08/10	−1,942%	0,000%	0,678%	0,433%	−0,099%	0,285%	0,2557
09	03/09/10–24/09/10	4,234%	26,919%	0,474%	0,003%	1,242%	0,159%	0,1927
10	01/10/10–29/10/10	1,193%	2,744%	0,043%	0,607%	0,316%	0,265%	0,1447
11	05/11/10–26/11/10	0,021%	14,084%	−1,486%	−1,392%	−0,935%	−1,384%	0,2428
12	03/12/10–31/12/10	3,181%	4,156%	1,098%	0,612%	0,466%	0,580%	0,4117
	Average	0,547%	4,953%	0,342%	−0,146%	−0,114%	−0,202%	0,472
	Std	2,40%	10,14%	1,63%	1,37%	1,09%	1,29%	0,281

Table 3
S & P 500, results.

Run	Out-of sample period	Naïve	MR	3 m mV	6 m mV	1y mV	Robust mV	t
01	01/01/10–29/01/10	−4,883%	16,303%	−5,708%	−1,494%	0,821%	−0,447%	0,340
02	05/02/10–26/02/10	4,033%	−1,849%	0,624%	−0,013%	2,884%	−0,221%	1,329
03	05/03/10–26/03/10	6,193%	9,975%	5,270%	3,325%	4,461%	3,839%	0,015
04	04/06/10–25/06/10	−1,185%	−4,247%	0,045%	0,513%	0,244%	0,290%	0,063
05	02/07/10–30/07/10	2,260%	−11,425%	4,965%	2,808%	1,436%	4,672%	0,135
06	03/09/10–24/09/10	8,554%	28,684%	4,537%	4,040%	5,140%	5,256%	0,186
07	01/10/10–29/10/10	3,302%	6,921%	3,158%	2,862%	2,538%	3,750%	0,289
08	05/11/10–26/11/10	1,330%	10,196%	1,255%	0,969%	0,780%	0,387%	0,080
09	03/12/10–31/12/10	−1,773%	12,894%	−0,987%	2,849%	1,312%	2,157%	0,003
	Average	1,981%	7,495%	1,462%	1,762%	2,180%	2,187%	0,271
	Std	3,92%	11,32%	3,32%	1,73%	1,61%	2,12%	0,390

optimal portfolios under stochastic dominance constraints was conducted by Dupacova and Kopa (2014). Moreover, Maillet et al. (2015) performed a worst-case minimum variance optimization with respect to alternative covariance matrix estimators.

Kim et al. (2013a) investigated robust models and fundamental factors in order to determine whether robust equity portfolios are more or less sensitive to factors than to individual assets' movements. Moving a step forward, Kim et al. (2014b) proposed robust modeling that allows the control of the level of exposure portfolios have in a factor. Moreover, in a study of composition of robust equity portfolios Kim et al. (2013b) inspected the properties of the selected assets. Kim et al. (2014a) also surveyed developments of robust worst-case optimization, including robust counterparts for value-at-risk and conditional value-at-risk problems. Kim et al. (2015) discussed robust optimization performance with focus on worst market state returns. Another robust worst-case approach within the best value-at-risk Sharpe ratio context is found in Deng et al. (2013).

A very comprehensive review of the 20-year old history of robust portfolio optimization is included in Kolm et al. (2014). Other research articles that summarize recent history and future trends of robust portfolio optimization are those of Fabozzi et al. (2007, 2010) and Scutellà and Recchia (2013), where the relation between robustness and convex risk measures is also studied. A thorough inspection of both theoretical and practical research in robust optimization was made by Ben-Tal et al. (2009).

Besides historical and theoretical reviews, useful guides for practitioners can be also found in Gorissen et al. (2015). In the robust multiobjective field, an effort to characterize the location of the robust Pareto frontier with respect to the corresponding original Pareto frontier using standard multiobjective optimization techniques was made by Fliege and Werner (2014). Finally, we also report other research attempts in the field of robust portfolio optimization, includ-