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An empirical study of chance-constrained portfolio selection model

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Abstract

This paper proposes an asymmetric approximation method for the chance-constrained portfolio selection model based on robust optimization techniques. We choose 30 assets from Chinese market to construct a portfolio and compare the performance of our model with Gauss approximation and Chebyshev approximation models. The experimental study shows that our model is able to put more weight on stocks with higher returns. Since, there is short-run persistence of the relative performance of the stocks, the portfolios constructed by our model can produce higher cumulative portfolio returns in the near future.

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Keywords: chance constraint, portfolio selection, robust optimization, asymmetry

1. Introduction

Consider a portfolio that consists of n assets. The current holdings in each asset are $w = (w_1, \ldots, w_n)$, The dollar amount transacted in each asset is specified by $x = (x_1, \ldots, x_n)$, $x_i > 0$ for buying, $x_i < 0$ for selling. After transactions, the adjusted portfolio $w + x$ is then held for a fixed period of time. At the end of that period, the return on asset i is the random variable a_i. The investor's goal is to maximize the expected wealth at the end of period, while satisfying a set of constraints on the portfolio. These constraints typically include:

The budget constraint: $\mathbf{1}^T x + \sum_{i=1}^n \phi(x_i) \leq 0$, where $\mathbf{1}$ is a vector with all entries equal to one, $\phi(x_i)$ is the transaction cost function for asset i,

$$\phi_i(x_i) = \begin{cases} \alpha_i^+ x_i, x_i \geq 0 \\ -\alpha_i^- x_i, x_i \leq 0 \end{cases}$$

Here α_i^+ and α_i^- are the cost rates associated with buying and selling asset i. Let $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$, Then $x_i = x_i^+ - x_i^-, \phi_i = \alpha_i^+ x_i^+ + \alpha_i^- x_i^-$.

They also impose the requirement that the end of period wealth W be larger than some undesired level W^{low} with a confidence level exceeding η, $\eta \in (1/2, 1)$. The shortfall risk constraint can be expressed as

$$Prob(W \geq W^{low}) \geq \eta. \tag{1}$$

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For ease of calculation, they assume that the returns, the random vector a, have a jointly Gaussian distribution, $a \sim N(\overline{a}, \Sigma)$, then the chance constraint (1) can be approximated as:

$$\Phi^{-1}(\eta)\|\Sigma^{1/2}(w + x)\| \leq \overline{a}^T(w + x) - W^{low}. \tag{2}$$

Diversification constraints: $w_i + x_i \leq \gamma_i \mathbf{1}^T(w + x), i = 1, \ldots, n$.
This portfolio selection problem is written as [1]:

$$\begin{aligned}
\text{(Gaussian)} \qquad \max \quad & \overline{a}^T(w + x^+ - x^-) \\
\text{s.t.} \quad & \mathbf{1}^T(x^+ - x^-) + \sum_{i=1}^{n}(\alpha_i^+ x_i^+ + \alpha_i^- x_i^-) \leq 0, \\
& w_i + x_i^+ - x_i^- \geq 0, i = 1, \ldots, n, \\
& w_i + x_i \leq \gamma_i \mathbf{1}^T(w + x), i = 1, \ldots, n, \\
& \Phi^{-1}(\eta)\|\Sigma^{1/2}(w + x^+ - x^-)\| \leq \overline{a}^T(w + x^+ - x^-) - W^{low}, \qquad (3) \\
& x_i^+ \geq 0, x_i^- \geq 0, i = 1, \ldots, n.
\end{aligned}$$

Many empirical studies show that portfolio returns are generally not normally distributed. Instead, their distributions are strongly asymmetric and fat-tailed ([2, 3]). In addition, the returns of portfolios involving credit derivatives can have extremely left-skewed distributions ([4]). For the portfolios with asymmetry returns, mean and variance cannot grasp all the characteristics of portfolio returns, therefore, Gauss approximation of chance constraints is inexact. Chen et al. [5] introduce an uncertainty set using the forward and backward deviation measures of bounded random variables to capture the distributional asymmetry. For this purpose, we approximate the chance constraint based on the work of Chen et al. [5]. We choose 30 representative assets from Chinese market to construct a portfolio and compare the performance of our model with Gauss approximation and Chebyshev approximation models.

Many scholars study the approximation of chance constraints, to our best knowledge, none makes a comparison of these approximation models empirically. This paper's main contribution is that we approximate the chance constraint based on the work of Chen et al. [5], which takes into consideration asymmetries in the distributions of returns; then, we compare the performance of our model with Gauss approximation and Chebyshev approximation models using Chinese real market data.

The remainder is organized as follows. We approximate the chance constraint based on robust optimization perspective in Section 2. Section 3 focuses on the empirical analysis of our model using Chinese real market data and presents computational results.

2. Approximation of chance-constrained portfolio selection model based on robust optimization

Robust optimization is one of the effective methods to deal with data uncertainty. It solves an optimization problem assuming that the uncertain input data belong to an uncertainty set, and finds the optimal solution if the uncertainties take their worst-case values within that uncertainty set. For the review of robust optimization, see [6] for details.

Consider the following optimization problem:

$$\begin{aligned}
\min \quad & \mathbf{c}^T\mathbf{x} \\
\text{s.t.} \quad & f(\mathbf{x}, \tilde{\mathbf{z}}) \leq 0, \\
& \mathbf{x} \in X,
\end{aligned} \tag{4}$$

where $\mathbf{x} \in \Re^N$ is the vector of decision variables, $\tilde{\mathbf{z}} \in \Re^N$ is a vector of uncertain factors, and the set X contains all the deterministic constraints.