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“On the (Ab)use of *Omega*?”

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A B S T R A C T

Several recent finance articles use the Omega measure (Keating and Shadwick, 2002), defined as a ratio of potential gains out of possible losses, for gauging the performance of funds or active strategies, in substitution of the traditional Sharpe ratio, with the arguments that return distributions are not Gaussian and volatility is not always the relevant risk metric. Other authors also use Omega for optimizing (non-linear) portfolios with important downside risk. However, we question in this article the relevance of such approaches. First, we show through a basic illustration that the Omega ratio is inconsistent with the Second-order Stochastic Dominance criterion. Furthermore, we observe that the trade-off between return and risk corresponding to the Omega measure, may be essentially influenced by the mean return. Next, we illustrate in static and dynamic frameworks that Omega-based optimal portfolios can be closely associated with classical optimization paradigms depending on the chosen threshold used in Omega. Finally, we present robustness checks on long-only asset and hedge fund databases, that confirm our results.

1. Introduction

A relatively wide stream of the financial economics literature focuses on performance measurement with two main motivations: the introduction of measures capturing stylized facts of financial returns such as asymmetry or non-Gaussian densities, among many others; the use of performance measures in empirical studies dealing with managed portfolio evaluation or asset allocation. The interest in this research field started with the seminal contribution of Sharpe in 1966. An increasing number of studies appeared in the following decades (see the survey by Caporin et al., 2014). One of the most studied topics is the performance evaluation of active management, which probably represents the main element promoting a renewed interest in performance measurement. Among the most recent contributions, we can quote Cherny and Madan (2009), Capocci (2009), Darolles et al. (2009), Jha et al. (2009), Jiang and Zhu (2009), Stavetski (2009), Zakamouline and Koekebakker (2009), Darolles and Gouriéroux (2010), Glawischnig and Sommersguter-Reichmann (2010), Billio et al. (2012), Cremers et al. (2013), Kapsos et al. (2014), Weng (2014), Billio et al. (2014) and Billio et al. (2015).

Performance evaluation has relevant implications from both a theoretical point of view, as it allows us to understand agent choices, and an empirical one, since practitioners are interested in ranking assets or managed portfolios according to a specific non-subjective *criterion*. As an example, financial advisors often rank mutual funds according to a specific performance measure. Moreover, when

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rankings produced by advisors are recognized as a reference by the investors, changes in the rankings might influence inflows and outflows (see Hendricks et al., 1993; Blake and Morey, 2000; Powell et al., 2002; Del Guercio and Tkac, 2008; Jagannathan et al., 2010).

A large number of performance measures have already been proposed and, as a consequence, related ranks can sensibly vary across different financial advisors, portfolio managers and investment institutions. The identification of the most appropriate performance measure depends on several elements. Among them, we cite the investors preferences and the properties or features of the analyzed asset/portfolio returns. Furthermore, the choice of the optimal performance measure, across a number of alternatives, also depends on the purpose of the analysis which might be, for instance, one of the following: an investment decision, the evaluation of managers abilities, the identification of management strategies and of their impact, either in terms of deviations from the benchmark or in terms of risk/return. Despite some known limitations, the Sharpe (1966) ratio is still considered as the reference performance measure. However, if we derive this ratio within a Markowitz framework, it shares the same drawbacks as the Mean–Variance model, where the representative investor is characterized by a quadratic utility function and/or the portfolio returns are assumed to be Gaussian.[1] Clearly, it is well established that financial returns are not Gaussian, also due to investment strategies based on derivatives with time-varying exposures and leverage effects. Furthermore, numerous theoretical articles and empirical studies show that it is unlikely that investors do not care about higher-order moments (*e.g.* Scott and Horwath, 1980; Golec and Tamarkin, 1998; Harvey and Siddique, 2000; Jondeau and Rockinger, 2006; Jurczenko and Maillet, 2006, and references herein).

As a consequence, both an incorrect assumption of Gaussianity and an under-estimation of the importance of higher-order moments for the investors, may lead to an underestimation of the portfolio total risk (see Fung and Hsieh, 2001; Lo, 2001; Mitchell and Pulvino, 2001; Kat and Brooks, 2002; Agarwal and Naik, 2004) and, consecutively, to biased investment rankings and financial decisions (a downward biased risk evaluation induces an upward biased Sharpe ratio). In this case, the Sharpe ratio does not completely reflect the attitude towards risk for all categories of investors. Moreover, the standard deviation equally weights positive and negative excess returns. In addition, it has been shown that volatility can be subject to manipulations (see Ingersoll et al., 2007). Using this quantity to evaluate the risk of assets with low liquidity can also be another issue (see Getmansky et al., 2004). Ultimately, the pertinence of using this performance measure relies heavily on the accuracy and stability of the first and second moment estimations (Merton, 1981; Engle and Bollerslev, 1986). Finally, the so-called "$\mu - \sigma$" Paradox illustrates that the Sharpe ratio is not consistent, in the sense of the Second-order Stochastic Dominance *criterion*[2] as introduced by Hadar and Russell (1969), when return distributions are not Gaussian (see Weston and Copeland, 1998; Hodges, 1998).

Based on these critics, academics and practitioners proposed alternative *criteria* that can be adapted to more complex settings than the Gaussian one. Among the various proposed measures to remedy these aforementioned shortcomings, the Gain–Loss Ratio was introduced by Bernardo and Ledoit (2000) as a special case of the Grinblatt and Titman (1989) Positive Period Weighted Measure of performance; it was then generalized later on (with a variable threshold instead of the nil reference — see below) first by Keating and Shadwick (2002) and, secondly, by Farinelli and Tibiletti (2008), Caporin et al. (2014), Bellini and Bernardino (2015) using variants of expected shortfall and expectiles. These performance measures were originally derived to fill the gap between model-based pricing and no-arbitrage pricing, and thus belong to the so-called "Good-Deal-Bound" literature (see Cochrane and Saa-Requejo, 2001; Cochrane, 2009; Biagini and Pinar, 2013). This literature mainly derives price bounds by precluding Stochastic Discount Factors which would create too attractive assets, with respect to their "Good-Deal" measure in the terminology of Cochrane (2009), and thereby inclining the no-arbitrage price bounds. Cochrane (2009) derives Good-Deal price bounds for option prices by restricting the Sharpe ratio. Černý (2003) generalizes this approach and made it well-applicable for skewed assets, whilst Cherny and Madan (2009) study the good properties that should exhibit any good performance measure. In this context, it appears that the Gain–Loss Ratio approach provides several significant advantages. A bounded ratio implies the absence of arbitrage opportunities. Another advantage, which might be the main reason for its success, results from its asset pricing model foundation. The Gain–Loss Ratio quantifies the attractiveness of an investment concerning a benchmark investor, *e.g.* a representative investor of a specified asset pricing model. Alongside the determination of price bounds on incomplete markets, the Gain–Loss Ratio can also be used to compare asset pricing models or to measure the performance of funds in an economically meaningful way, *i.e.* concerning a benchmark Stochastic Discount Factor. In addition to the stated theoretical foundation and the preclusion of arbitrage opportunities, the Gain–Loss Ratio has many other desirable properties. It is not only very intuitive, but it also fulfills many requirements of a good performance measure, *e.g.* all conditions of an "acceptability index" as put forward by Cherny and Madan (2009).[3]

Nevertheless, some authors recently report a first notable drawback: in many standard models, *e.g.* the Black–Scholes model, the best Gain–Loss Ratio becomes infinity. Because of this shortcoming, Biagini and Pinar (2013) conclude that the "Gain–Loss Ratio is a poor performance measure..." (*sic*). One might argue that the Gain–Loss Ratio is only constructed for discrete probability space corresponding to real asset returns, but even in this case, the strong dependence on the specific discretization can be seen as a severe drawback. Voelzke (2015) recently attempted to solve this issue by proposing the so-called Substantial Gain–Loss Ratio, that allows us to work in continuous probability spaces without losing the positive properties of the Gain–Loss Ratio.

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[1] The Gaussianity assumption might be relaxed when considering, for instance, a Student-t density characterized by a specific degree of freedom.

[2] The following simple illustration shows that the Sharpe ratio does not even comply with the First-order Stochastic Dominance *criterion*: if we compare an investment A with potential outcomes such as {−1%;1%;5%} with probabilities {.1;.45;.45}, to an investment B with potential outcomes of {−1%;1%;3%} with (the same) probabilities {.1;.45;.45}, then the Sharpe ratio of B is higher than those of A (*i.e.* 1.30 *versus* 1.16) whilst, obviously, A dominates B according to the First-order Stochastic Dominance *criterion* — see other examples in Hodges (1998), Weston and Copeland (1998); see also Homm and Pigorsch (2012a) and (2012b) for further discussion.

[3] Note, however, that Cherny and Madan (2009) use a non-standard definition of the Gain/loss ratio, with a difference in the numerator: they define the ratio as the expected return over the expected loss (for positive expected returns), which leads to the consistency of this ratio with the SSD criterion.